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Description of Research Seminars

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Participants:	UGOE UNIKL DTU UOA UC
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Research Seminar

offered by Timm Treskatis (University of Canterbury)

in July 2014,

in Göttingen, Germany

Subject: A Trust-Region SQP Method for the Numerical Approximation of Viscoplastic Fluid Flow

Problem: Viscoplastic materials arise in a broad range of industrial contexts: food liquids (tomato sauce, jam, mayonnaise), consumer goods (toothpaste, hair gel, paint) and fluids of great relevance in engineering (cement, waxy crude oil and certain lubricants) all exhibit an interesting physical phenomenon. Whenever the stress falls below a critical value, the so-called yield stress, these fluids appear to segue from the viscous motion of a liquid to the rigid behaviour of a plastic solid, and vice versa. Mathematically, this feature results in highly nonlinear and nonsmooth models for such materials, which yields a system of variational inequalities that extends the classical Navier-Stokes equations.

Smoothing and regularisation are very common to avoid the analytical and computational challenges of nonsmooth problems. For many crucial applications, however, such approximations of the actual material behaviour are not desirable or even nonsensical. Meanwhile, existing genuinely nonsmooth approaches that employ augmented Lagrangian techniques suffer from poor convergence, and the resulting computational cost of such simulations easily becomes prohibitively high.

To develop an alternative approach that allows for both an exact treatment and efficient numerical solutions, we initially consider viscoplastic duct flows of so-called Bingham and Herschel-Bulkley fluids. These are prototypical problems for a much broader class of convex optimisation problems and therefore of interest for researchers from image processing, machine learning and many other disciplines as well.

Main Results: The results presented in this seminar comprise both the analytical formulation and numerical algorithms.

Firstly, we derive an equivalent formulation which possesses higher regularity and which is therefore amenable to a larger class of numerical schemes.

While the optimisation problem is initially given as an unconstrained nonsmooth minimisation problem, it may be transformed into a constrained problem by introducing auxiliary variables. We can then study a dual problem instead. Physically, this transformation corresponds to a change of variables from a flow velocity (primal variable) to a stress (dual variable). In contrast to the non-differentiable primal problem, the dual optimality system is at least Newton-differentiable, depending on the underlying constitutive model.

Secondly, as opposed to contrary claims in the literature, we can establish well-posedness of the problem as well as existence of solutions, with a unique primal solution.

Thirdly, we study (semismooth) methods of Newton-type. By incorporating a damping strategy through trust-region constraints in every iteration, we can show that such algorithms converge to a solution of the problem. Numerical experiments confirm that this approach outperforms state-of-the-art augmented Lagrangian methods. The greatest gains in computational efficiency occur for problems with the highest degree of nonlinearity.

Participants: Researchers and students from the University of Göttingen

Publication: T Treskatis, M A Moyers-González, C J Price: *A Trust-Region SQP Method for the Numerical Approximation of Viscoplastic Fluid Flow.* Submitted (2014). Preprint: `http://arxiv.org/abs/1504.08057`