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Optimization and its **A**pplications in **L**earning and **I**ndustry (OptALI)

IRSES

Ongoing Deliverable D1.2

Description of Research Seminars

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Participants:	UGOE UNIKL DTU UOA UC
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Research Seminar

offered by Sönke Behrends (University of Göttingen)

in February 2015,

in Christchurch, New Zealand

Subject: Minimization of a polynomial over the integers

Problem: Given a polynomial in several variables, we consider the problem to find the minimum value it attains over the integer lattice. A famous result from complexity theory tells us that this problem cannot be solved in full generality. In fact, in 1900 at the International Congress of Mathematics, David Hilbert presented his list of 23 problems in mathematics – and problem 10 concerned whether there is an *efficient method* that solves general Diophantine equations, i.e., if there is an algorithm that, given a polynomial f in n variables with integer coefficients, determines if there is an integer solution to the equation $f(x_1, \ldots, x_n) = 0$. Seventy years later, in 1970, Matiyasevich answered the question in the negative: No such algorithm can exist. This has an immediate consequence for integer optimization: Suppose we know an algorithm that minimizes arbirtrary polynomials f in n variables over the integer lattice. Given a Diophantine equation $g(x_1, \ldots, x_n) = 0$, we can decide the existence of solutions by minimizing g^2 over the integer lattice – a contradiction to Matiyasevich's result. Thus, to make the make the problem tractable, we have to consider promising subclasses. Applications are, e.g., the Closest Vector Problem, portfolio optimization, as well as solving Diophantine equations.

Main Results: Such a tractable case is present if the polynomial grows "fast enough", that is, if the leading form of the polynomial - given by its highest order terms - attains positive values only (except at 0). Once we know this condition holds, we can compute the radius of a ball that must contain all optimal integer solutions. We could now simply enumerate all solutions and compare their function values to find the optimal ones. However, this soon gets prohibitive for more than a few variables or a large radius. A technique known in the literature as branch and bound in conjunction with tight lower bounds can help to speed up the solution process. To this end, we present a new class of lower bounds and some results on random instances. In this talk we also address the computational techniques involved as well as the corresponding software implementations.

This is joint work with Ruth Hübner and Anita Schöbel.

Participants: Researchers and students from the University of Canterbury.

Publication: -