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Optimization and its **A**pplications in **L**earning and **I**ndustry (OptALI)

IRSES

Ongoing Deliverable D1.2

Description of Research Seminars

Start date of the Workpackage:	December 2010
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Participants:	UGOE UNIKL DTU UOA UC
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Research Seminar

offered by Morten Tiedemann (ES-UGOE-8)

in January 2014,

in Christchurch, New Zealand

Subject: The Online Knapsack Problem with Incremental Capacity - Results and Future Research

Problem: We consider an online knapsack problem with incremental capacity. In each time period, a set of requests (items), each with a specific weight and value, is revealed and, without knowledge of future requests, it has to be decided which of these requests to accept (i.e., pack into the knapsack). Furthermore, the knapsack capacity changes dynamically over time, i.e., the capacity is not fully available from the start but a constant additional amount of capacity becomes available in each time period. Hence, if $k \geq 1$ denotes the amount of additional capacity that becomes available in each time period, the available capacity in time period $i \in \{1, \ldots, T\}$ is $k \cdot i$ minus the total weight of all requests that have been accepted in time periods 1 to $i-1$, where T denotes the total number of time periods considered. The goal is to maximize the overall value of the accepted requests while respecting the capacity constraint in each time period.

In contrast to the basic online knapsack problem, for which no competitive algorithms exist, the setting of incremental capacity facilitates the development of competitive algorithms. The problem is investigated by competitive analysis for deterministic and randomized online algorithms.

Main Results: We discuss a model for the online knapsack problem with incremental capacity. For the case of unit weight requests and unit incremental capacity (i.e., one additional unit of capacity becoming available in each

time period), we give a deterministic T-competitive online algorithm and a matching lower bound on the competitive ratio of any deterministic online algorithm. For unit weights and k-incremental capacity (where $k \geq 2$ additional units of capacity become available in each time period), a deterministic $^{Tk}/_{(2k-2)}$-competitive algorithm for $T \geq 2k-2$ and a deterministic $^{Tk}/_{\left(k+\left\lfloor \frac{T}{2} \right\rfloor\right)}$-competitive algorithm for $T < 2k-2$ are proposed. Moreover, a randomized algorithm with a competitive ratio of $^{(T+1)}/_{2}$ is developed for this setting, matching the lower bound for any randomized algorithm.

For the case that general nonnegative weights are allowed, we show that no competitive online algorithm exists for the problem. However, for limited weights in $\{1, \ldots, k\}$ and k-incremental capacity, we present a competitive deterministic online algorithm and a lower bound on the competitive ratio of any deterministic online algorithm that approaches the competitive ratio of the proposed algorithm for $k \to \infty$.

Participants: Students and researchers from the University of Canterbury

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