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Ongoing Deliverable D1.2

Ellipsoid Bounds for Convex Quadratic Integer Programming

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Research Seminar

offered by Ruth Hübner (ES-UGOE-5)

in March 2013,

in Auckland, New Zealand

Subject: Ellipsoid Bounds for Convex Quadratic Integer Programming

Problem: We consider the problem of minimizing a strictly convex quadratic function $q(x) := x^\top Qx + v^\top x + c$ (where Q is symmetric and positive definite) over the integer lattice. Since a closed formula for the optimal solution of the continuous relaxation of this problem exists and the integer problem is NP-hard it seems promissing to identify special cases - specified by the matrix Q - that guarantee that rounding the optimal solution to the continuous relaxation (by "rounding" we mean rounding each component of this solution either up or down and hence gain up to 2^n rounded points) yields an optimal solution to the integer problem. In general this is of course not going to work, but for example if Q is the identity matrix this is a permitted way of solving the problem.

Considering the level sets of q we see that they are ellipsoids centered in the optimal solution to the continuous relaxation, whose shape only depend on Q. Using this information it is now possible to identify matrices Q - and therefore objective functions q - that allow our approach of rounding an the optimal continuous solution.

In a general strictly convex quadratic problem we can on the other hand not expect Q to have this property. Therefore we underestimate the given function by another strictly convex quadratic function that has this property - and can therefore easily be minimized over the integer lattice - to obtain lower bounds for a branch-and-bound-approach.

Main Results: Since different shapes of ellipsoids (for example axisparallel ones) guarantee that an optimal integer solution is among the rounded points of the optimal continuous relaxation and since on the other hand a given ellipsoid can be underestimated by a lot of different asxisparallel ellipsoids we obtain an optimization problem of finding the best axisparallel underestimation and different bounds depending on which shape of ellipsoids we require.

Solving the optimization problem to find (for example) the best axisparallel approximation shows that the best approximation does not only depend on the shape of the given ellipsoid but also on the center - which is the continuous optimal solution. The same is true for the question which shape (e.g. axisparallel) provides the best lower bound. This makes things harder in a branch-and-bound-approach since it is time-consuming to calculate the approximation and we would therefore like to avoid doing it in every node. On the other hand the continuous optimal solution changes in each node. This leads us to a worst-case- and an average-cases-analysis to find approximations that are good in the worst-case or (in a branch-and-bound-approach with usually a lot of nodes even more fitting) in the average case.

Some first computational results compare those bounds with the already used ones and are promising.

Participants: students and researchers from UOA.

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