FP7-PEOPLE-2009-IRSES:
Project ID 246647

Optimization and its Applications in Learning and Industry (OptALI)

IRSES

Ongoing Deliverable D1.2

Ellipsoid Bounds for Convex Quadratic Integer Programming

Start date of the Workpackage:	December 2010
Duration:	48 months
Due date of deliverable:	November 2014
Actual submission date:	April 2013
Participants:	UGOE UNIKL DTU UOA UC
Author of deliverable:	R. Hübner (r.huebner@math.uni-goettingen.de)

Research Seminar

offered by Ruth Hübner (ES-UGOE-5)

in January 2013,

in Christchurch, New Zealand

Subject: Integer Nonlinear Optimization - Applications of the Rounding Property

Problem: Since there are many integer nonlinear problems where solving the continuous relaxation is significantly easier than solving the original IP the approach of solving the continuous relaxation and rounding the gained optimal solution to an integer point seems tempting. By "rounding" we mean rounding each component of a point $x \in \mathbb{R}^n$ either up or down. This means that for each point x we get up to 2^n "rounded points" and the question is: Is one of them optimal for the IP? In general they do not even have to be feasible but even if we consider an unrestricted optimization problems rounding an optimal continuous solution will not provide us with an optimal solution of the IP in general. But there are problems that have this kind of "Rounding Property" (RP) and can be solved by solving their continuous relaxation and then testing the up to 2^n points we get by rounding this solution. This is efficient, if the continuous relaxation can be solved efficiently and if n is not to large.

The main problem is therefore: Given an IP, how can we decide whether it has the Rounding Property or not? To answer this question we choose a level set approach: We identify geometric shapes that guarantee, if the intersection of the level sets of the objective function with the feasible region have this shape, that the problem has the Rounding Property.

To compare this approach of solving integer nonlinear optimization problems with other approaches we need to identify problems for which we can guarantee the Rounding Property. Also we are intersted in how we can generalize this approach to mixed integer problems and how it changes if we try to solve robust optimization problems.

Main Results: A good class of problems to apply the concept of Rounding Property seems to be the class of location problems, since the dimension is often only 2 - which means we have to compare 4 rounded points, the level sets are often known and in many applications there are no further constraints. Furthermore many location problems are well-researched which means that efficient methods for solving the continuous relaxation are often provided. On the other hand we have the impression that adding an additional integrality constraint to location problems is not yet that well researched. Of course not all location have the rounding property - for example the Weber-Problem with Euclidean distance which does not have the RP in general. On the other hand there are some examples where the RP can be proven easily - for example the Weber-Problem with rectangular distance or the barycenter problem.

If we adopt our approach to find strictly robust optimal solutions, i.e. to find a solution that is feasible in each scenario and minimizes the worst-case objective function. The feasible region is the intersection of the feasible regions for each scenario and the level sets of the worst-case objective function ($f(x) = \sup_{\xi \in U} f(x, \xi)$) are the intersection of the level sets for each scenario. This raises the question: Under which assumptions on the scenario set U and the feasible regions/ level sets for each scenario does the arising robust optimization problem have the RP? This boils down to identifing the geometric shape of the intersections of the feasible regions and the level sets.

To adopt our results to mixed integer nonlinear optimization problems we define that a MINLP has the RP, if optimal integer variables can be found by rounding the corresponding variables of an optimal solution to the continuous relaxation. The remaining continuous variables are then found by solving a lower dimensional continuous optimization problem. It is rather easy to see that the results obtained so far can not just be carried over to this new situation.

Participants: students and researchers from UC.

Publication: Hübner, R., Schöbel, A., When is rounding allowed in integer nonlinear optimization?. Submitted. (2012)