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Optimization and its Applications in Learning and Industry (OptALI)

IRSES

Ongoing Deliverable D1.2

Description of Research Seminar: Shortest Paths, Bottleneck Paths, and Matrix Multiplication

Start date of the Workpackage:	December 2010
Duration:	48 months
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Actual submission date:	December 2013
Participants:	UGOE UNIKL DTU UOA UC
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Research Seminar

offered by Tong-Wook Shinn (University of Canterbury)

in November 2013,

in Goettingen

Subject: Shortest Paths, Bottleneck Paths, and Matrix Multiplication

Problem: We introduce a new problem in graph theory whereby we calculate the shortest paths for all possible flow amounts between pairs of vertices. We call this problem the Shortest Paths for All Flows (SP-AF) problem. On a graph with n vertices and m edges, there are $O(m)$ possible distinct edge capacities. We refer to the distinct edge capacities as maximal flows. Then the SP-AF problem is to compute the shortest paths for all maximal flows. We focus on the All Pairs SP-AF (APSP-AF) problem, where we compute the shortest paths for all flows for all possible pairs of vertices on the graph.

Main Results: Let t be the number of distinct edge capacities. Then it is straightforward to solve the APSP-AF problem in $O(tn^3)$ time complexity by using Floyd's algorithm to solve the APSP problem for all t maximal flows. We show that on graphs with unit edge costs, the problem can be solved in $O(\sqrt{t}n^3)$ time, and on graphs with integer edge costs bounded by c, the problem can be solved in $O(\sqrt{t}c^2n^3)$ time. We can utilize faster matrix multiplication over a ring to give better time bounds of $O(\sqrt{t}n^{2.844})$ and $O(\sqrt{t}c^{1.844}n^{2.844})$, respectively.

Participants: Researchers from UGOE.

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