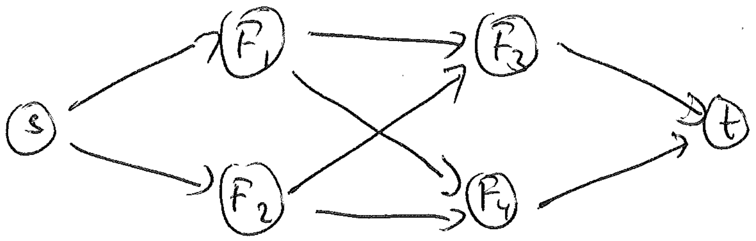
Crew Scheduling

Flight Schedule

F_1	CHC	AKL	6.40	8.00	A_1
F_2	WLG	AKL	7.15	8.15	A_2
F_3	AKL	CHC	8.45	10.05	A_2
F_4	AKL	ZQN	8.45	10.35	A_1

Minimise total time. Observe need 2 pilots and need 30 min ground time for A/C + crew



4 feasible sequences

$s - F_1 - F_3 - t$	205
$s - F_1 - F_4 - t$	235
$s - F_2 - F_3 - t$	170
$s - F_2 - F_4 - t$	200

Set partitioning problem

$$\begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} 205 & 235 & 170 & 200 \end{array} \end{array} X = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Optimal solutions $\boxed{x_1 = x_4 = 1}$ or $x_2 = x_3 = 1$

Uncertainty in flying times

Scenario 1 : F_2 takes 75 instead of 60 minutes $p_1 = \frac{1}{3}$

Scenario 2 : F_4 takes 140 instead of 110 mins $p_2 = \frac{1}{3}$

Scenario 3 : F_1 takes 110 instead of 80 mins

+ F_2 takes 65 instead of 60 mins $p_3 = \frac{1}{3}$

Stochastic programming

$$\min c^T x + Q(x) \qquad \left(= \sum_{\omega \in \Omega} p_\omega Q(x, \omega) \right)$$
$$\text{s.t.} \quad Ax = e$$
$$x \in \{0,1\}^4$$

↓ Recourse cost in sc. ω
Adjust arrival / dep times

Recourse action

S1 : $F_2 \to$ Delay F_4 by 15 minutes (crew)
$F_2 \to$ Delay F_3 by 15 minutes (AC)

S2 : No change

S3 : $F_1 \to$ Delay F_4 by 15 minutes (AC)
$F_1 \to$ Delay F_3 by 15 minutes (crew)
$F_2 \to$ Delay F_3 by 5 minutes (crew)
$F_2 \to$ Delay F_4 by 5 minutes (crew)

Crew induced delay

$F_1 - F_3$ $\quad \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 0 + \frac{1}{3}(15 + 5) = \frac{20}{3}$

$F_2 - F_4$ $\quad \frac{1}{3} \cdot 15 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 5 = \frac{20}{3}$

Forbid $F_2 - F_4$

$$\begin{array}{cccc} 205 & 235 & 170 & \cancel{200} \\ 1 & 1 & 0 & \cancel{0} \\ 0 & 0 & 1 & \cancel{1} \\ 1 & 0 & 1 & \cancel{0} \\ 0 & 1 & 0 & \cancel{1} \end{array} \quad x \geq \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}$$

Optimal solution $x_2 = x_3 = 1$

Recourse cost

S_1 Delay F_3 by 15 minutes (AC/crew)
S_2 No change
S_3 Delay F_4 by 15 minutes (AC/crew)
Delay F_3 by 5 minutes (AC/crew)

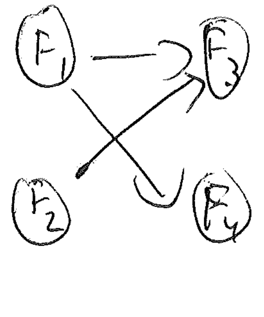
No aircraft changes hence no crew induced delay
$Q(x) = 0$ STOP

Robust Optimization

	S0	S1	S2	S3	S0	S1	S2	S3	
F_1	6.40	6:40	6.40	6.40	8.00	8.00	8.00	8:30	A_1
F_2	7.15	7.15	7.15	7.15	8.15	8.30	8.15	8:20	A_2
F_3	8.45	9:00	8.45	8:50	10.05	10:20	10.05	10.10	A_2
F_4	8.45	8.45	8.45	9:00	10.35	10:35	11:05	10:50	A_1

Delays caused by AC

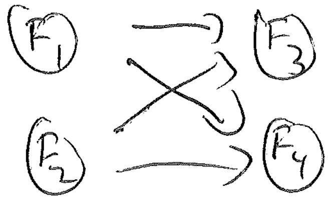
S1



solution $x_2 = x_3 = 1$

Cost 235 + 185

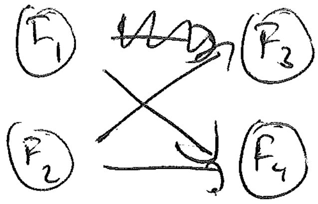
Scenario 2:



$x_2 = x_3 = 1$ $\quad$ $265 + 170$

$x_1 = x_4 = 1$ $\quad$ $210 + 215$

Scenario 3:



solution

$x_2 = x_3 = 1$

Cost $175 + 250$

Only 1 solution feasible in all scenarios

<u>Multiobjective</u>

Use crew induced delay as second objective

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{matrix} 205, & 235 & 170 & 200 \\ 20/3 & 0 & 0 & 20/3 \end{matrix}$$

Only 1 efficient solution $x_2 = x_3 = 1$

Talk 1 Bi-objective Model

Talk 2 Comparison with Stochastic

So for aircraft assignment given
$x_1 = x_4 = 1$ contains 2 aircraft changes,
$x_2 = x_3 = 1$ None.

Can achieve the same by reassigning Aircraft
$F_3 \to A_1$, $F_4 \to A_2$

Talk 3 Iterative scheduling

Now consider Captain and First officer

a)	captain	$x_2 = x_3 = 1$	0	aircraft changes
	FO	$x_1 = x_4 = 1$	2	
b)	Captain	$x_1 = x_4 = 1$	2	aircraft changes
	FO	$x_2 = x_3 = 1$	0	
c)	captain = FO	$x_1 = x_4 = 1$	4	"
d)	captain = FO	$x_2 = x_3 = 1$	0	"

Talk 4 Unit crewing