FP7-PEOPLE-2009-IRSES:
Project ID 246647

Optimization and its **A**pplications in **L**earning and **I**ndustry (OptALI)

IRSES

Ongoing Deliverable D1.2

Description of Research Seminar: The Linear Bi-Objective Multi-Commodity Minimum Cost Flow Problem

Start date of the Workpackage:	December 2010
Duration:	48 months
Due date of deliverable:	November 2014
Actual submission date:	January 2012
Participants:	UGOE UNIKL DTU UOA UC
Author of deliverable:	S. Moradi (siamak.ind@gmail.com)

Research Seminar

offered by Siamak Moradi (UOA)

in August 2012,

in Kaiserslautern, Germany

Subject: The Linear Bi-Objective Multi-Commodity Minimum Cost Flow Problem.

Problem: Network flow models are in general used to model a variety of real-world decision making problems in a wide range of areas. In many application contexts, there are several objectives as well as several different commodities that have to be taken into account. Thus, multi-objective multi-commodity flow models are appropriate for modelling real world decision making situations in the field of network optimization. The bi-objective multi-commodity minimum cost flow problem (*BMCMCF*) can be defined as a network problem where we want to send several commodities from their source nodes to their sink nodes. Individual commodities share common arcs and compete for the capacity of these arcs. Each commodity on each arc has a pair of unit flow costs, and the total sum of these costs should be minimized as the objectives of the problem. We seek for a complete set of efficient solutions.

Main Results: Modelling the *BMCMCF* problem as a linear program permits us utilizing well known bi-objective methods. We use two methods to solve the problem, a bi-objective version of the parametric simplex method and the dichotomic method. The bi-objective simplex method is initially started by obtaining a solution and related basis which is minimal with respect to the first objective component. Then the method iteratively continues finding entering variables to the basis with maximum ratio of improvement of the second objective and deterioration of the first objective. This continues until all efficient solutions are

obtained. An alternative to the parametric approach is the dichotomic approach. The method involves solving several single objective problems in weighted sum formulation. In this method, the initial points are obtained by solving the problem with respect to the first and second objective. Next, weights are chosen to obtain a supported non-dominated point and corresponding efficient solution that has the maximal distance to the straight line connecting the the images of the two initial points. Whenever the image of the obtained efficient solution of such a problem does not lie on the line connecting the images of the former solutions, two new sub-problems can be formulated. Otherwise, there are no more extreme supported points between them, so the current subproblem does not have to be split up further. The dichotomic method stops if no new weighted sum problems have to be solved, which means a complete set of the extreme supported efficient solutions is obtained.

We investigate the performance of the methods on different sets of bi-objective network instances with several commodities. The following observations are made from the implementation of the methods: By increasing the number of commodities the CPU running time increases significantly for both of the methods. The number of non-dominated extreme points and consequently the average CPU time increases by increasing the number of nodes or by increasing the number of arcs. The parametric method solves the problems in less CPU time than the dichotomic method for medium density problems. For high density network examples the dichotomic method preforms better than the parametric method.

Participants: students and researchers from UNIKL.

Publication: Siamak Moradi, Matthias Ehrgott, Andrea Raith "The Linear Bi-Objective Multi-Commodity Minimum Cost Flow Problem", 46^{th} Annual ORSNZ Conference (Dec, 2012), Wellington, New Zealand.