k Distance Recoverable Robustness

Christina Büsing

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k-Distance Recoverable Robustness

Given:
- Set U, feasible solutions $\mathcal{F} \subseteq 2^U$, cost $c^D : U \rightarrow \mathbb{R}$.
- Scenario set $\mathcal{S}$: S defines $c^S : U \rightarrow \mathbb{R}$ and $\mathcal{F}^S \subseteq \mathcal{F}$
- Recovery by distance parameters $k, \ell \in \mathbb{N}$

$$\mathcal{F}^S(F) = \{F' \in \mathcal{F}^S \mid |F' \backslash F| \leq k,\ |F \backslash F'| \leq \ell\}$$

Find: $F \in \mathcal{F}$ with minimum cost

$$c(F) = c^D(F) + \max_{S \in \mathcal{S}} \min_{F^S \in \mathcal{F}^S(F)} c^S(F^S)$$

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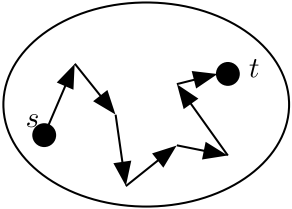
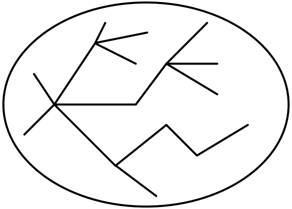


SP, $k = 2$



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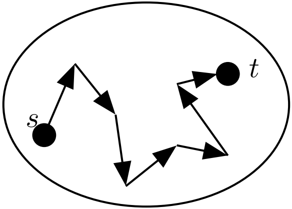

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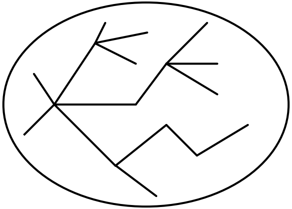

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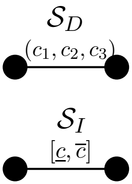

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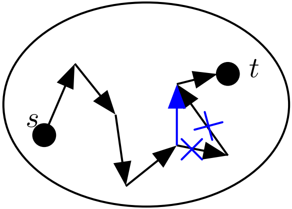

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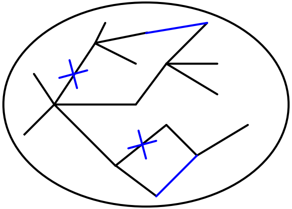
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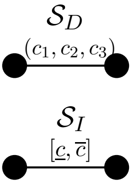

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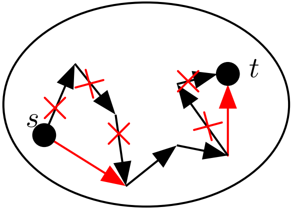

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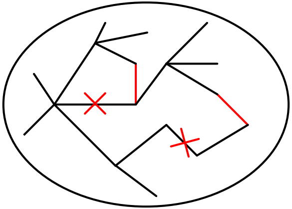

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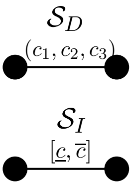

Related Work

Robustness

- Introduction for linear programs [Soyster '73]
- Combinatorial optimization discrete scenarios [Kouvelis & Yu '97, Aissi et al. '05]
- Polynomially solvable for Γ-scenarios [Bertsimas & Sim '03]

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Recoverable Robustness

- Adaptive robustness and complexity [Ben-Tal et al. '04]
- Recoverable robustness [Liebchen et al. '09]
- Two-stage robustness [Golovin et al. '06, Gupta et al. '10]
- α-robustness [Hassin & Rubenstein '02, Fujita et al. '10]

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Applications of Recoverable Robustness

- Public transportation
- Telecommunication

Outline

Complexity and Combinatorial Properties

Exact Algorithms

Growing into Application

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- weakly/strongly $\mathbf{NP}$-hard, polynomial solvable
- structure and properties of optimal solutions

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- cutting planes

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Growing into Application

- train classification
- bandwidth packing

k-Dist-RR Shortest Path Problem

Given: $G = (V, A)$, first-stage cost $c^D \colon A \to \mathbb{N}$, scenarios $\mathcal{S}$, recovery parameter k, recovery

$$\mathcal{P}^k(p) = \{p' \in \mathcal{P} \mid |p' \backslash p| \leq k\}$$

Find: path p with minimal total cost

$$c(p) = c^D(p) + \max_{S \in \mathcal{S}} \min_{p' \in \mathcal{P}^k(p)} c^S(p')$$

Results:

- simple paths, k const., $\mathcal{S}_I$: not approximable [2-dis. path]
- $\mathcal{S}_D$, c^D obey $\alpha_{[0,1]}$-deviation: 1.5 approximation (best poss.)
- general paths, k const., $\mathcal{S}_I$: polynomial solvable
- general paths, k input, $\mathcal{S}_I$: not approximable [3SAT]

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Nominal Problem

$$\min cx$$

$$a_i x \leq b_i \quad i \in [m]$$

$$x \in \{0,1\}^n$$

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k-Dist RR ILP

$$\min c^0 x + \max_{S \in \mathcal{S}} c^S x^S$$

$$a_i x \leq b_i$$

$$a_i^S x^S \leq b_i \quad S \in \mathcal{S}$$

$$\sum_i \max\{x_i^S - x_i, 0\} \leq k \quad S \in \mathcal{S}$$

$$\sum_i \max\{x_i - x_i^S, 0\} \leq \ell \quad S \in \mathcal{S}$$

$$x^S, x \in \{0,1\}^n$$

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Drawbacks

- exponential scenario sets
- size

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New Formulations

- robust cutting planes
- compact formulations, e.g., $\mathcal{S}_\Gamma$

Complexity and Combinatorial Properties

- weakly/strongly $\mathbf{NP}$-hard, polynomial solvable
- structure and properties of optimal solutions

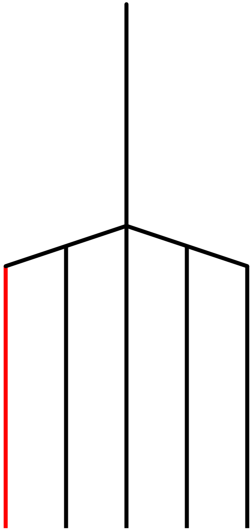
Exact Algorithms

- compact ILP formulations
- cutting planes

Growing into Application

- train classification
- bandwith packing

Train Classification



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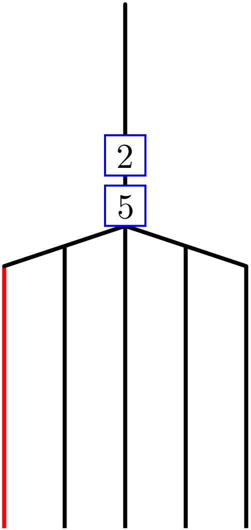

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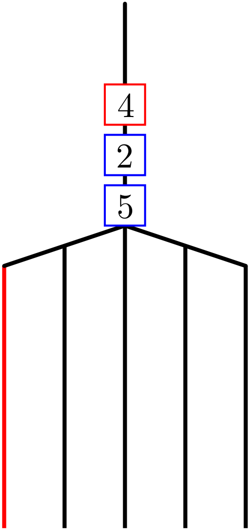

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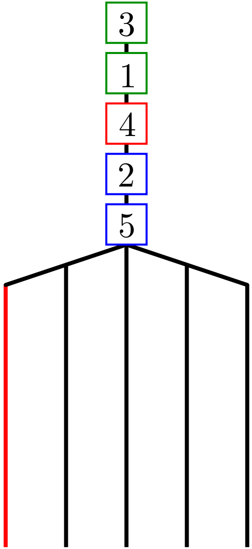

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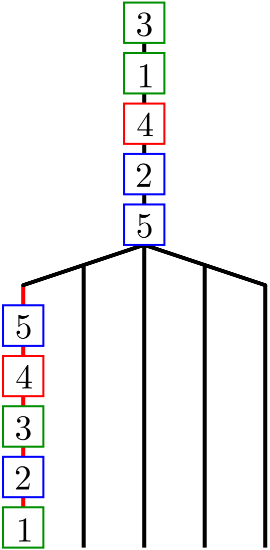

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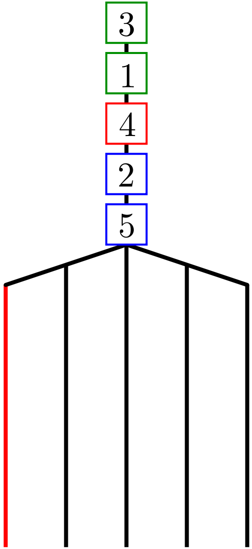

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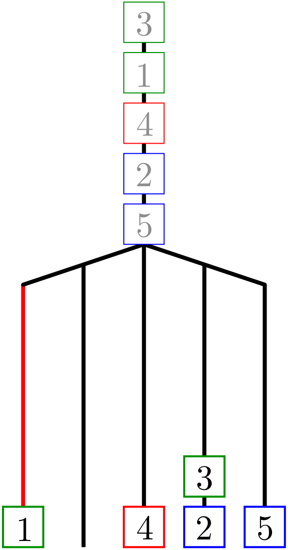

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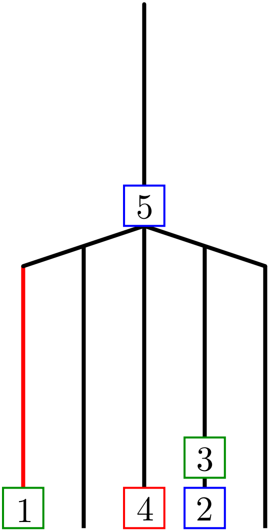

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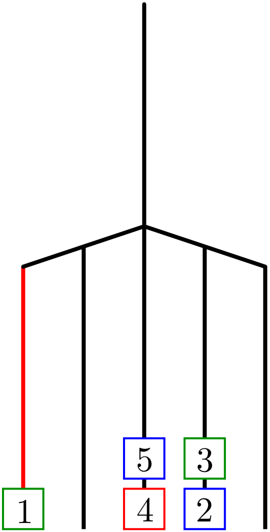

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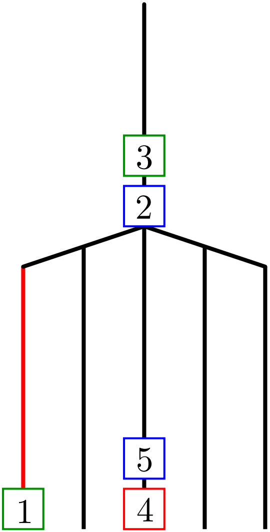

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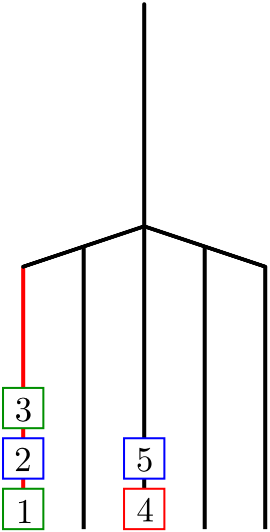

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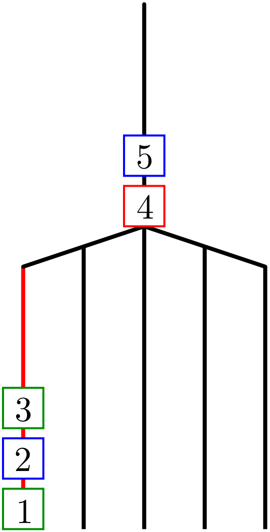

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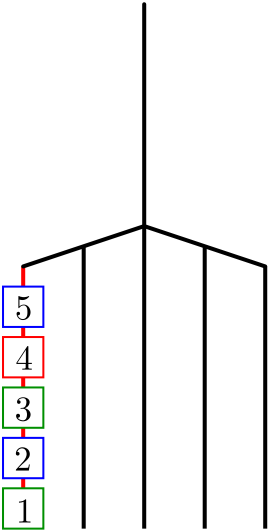
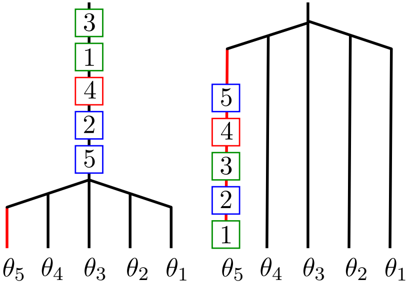




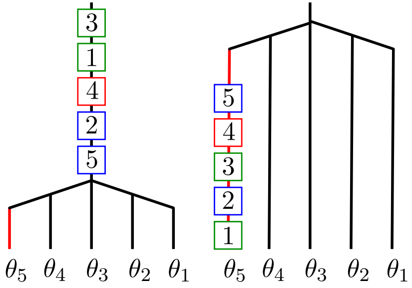
0	1	0	0	0	0	1
1	1	0	0	0	1	2
1	1	0	0	0	1	3
2	1	0	0	1	0	4
3	1	0	0	1	1	5
	θ_5	θ_4	θ_3	θ_2	θ_1	

Given:

- Incoming trains $T_1, \ldots, T_\ell$

Find:

- Schedule with minimum number of sorting steps



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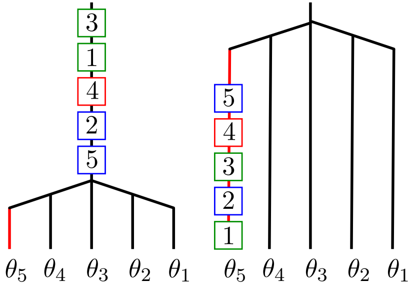

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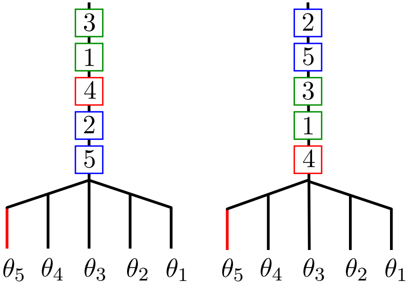
Given:

- Incoming trains $T_1, \ldots, T_\ell$
- Delayed trains

Find:

- Schedule with minimum number of sorting steps



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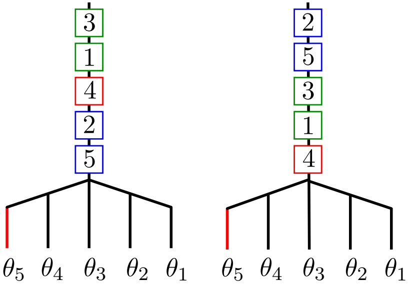

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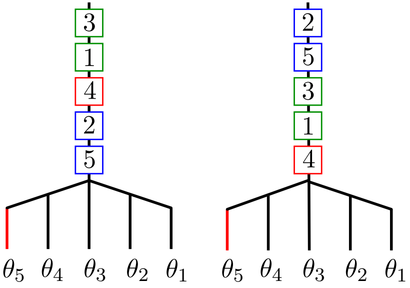
- Additional sorting steps k

Find:

- Schedule with minimum number of sorting steps



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Given:

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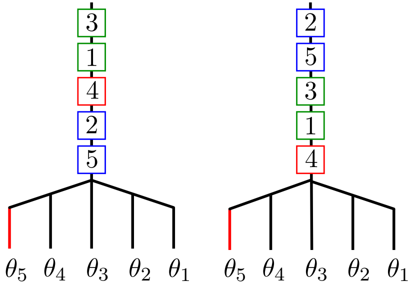
- Additional sorting steps k
- Offset $p - 1$

Find:

- Schedule with minimum number of sorting steps



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θ_5	θ_4	θ_3	θ_x	θ_p	θ_2	θ_1	

$\longleftarrow h-p \longrightarrow$ $\longleftarrow k \longrightarrow$ $\longleftarrow p \longrightarrow$

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Find:

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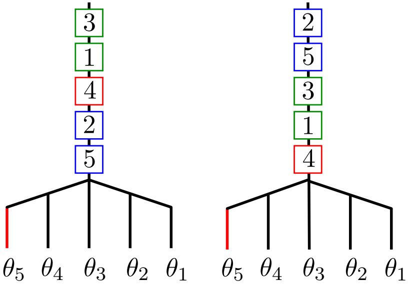

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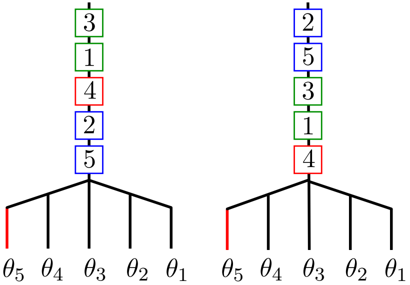

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$\overleftrightarrow{h-p}$ $\overleftrightarrow{k}$ $\overleftrightarrow{p}$

Given:

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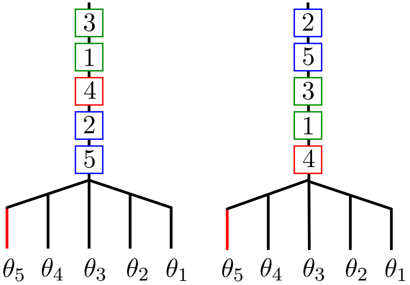
Find:

- Schedule with minimum number of sorting steps
- recoverable



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$h-p$ (columns θ_5, θ_4, θ_3); k (column θ_x); p (columns θ_p, θ_2, θ_1)

Given:

- Incoming trains $T_1, \ldots, T_\ell$
- Delayed trains
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- Offset $p-1$

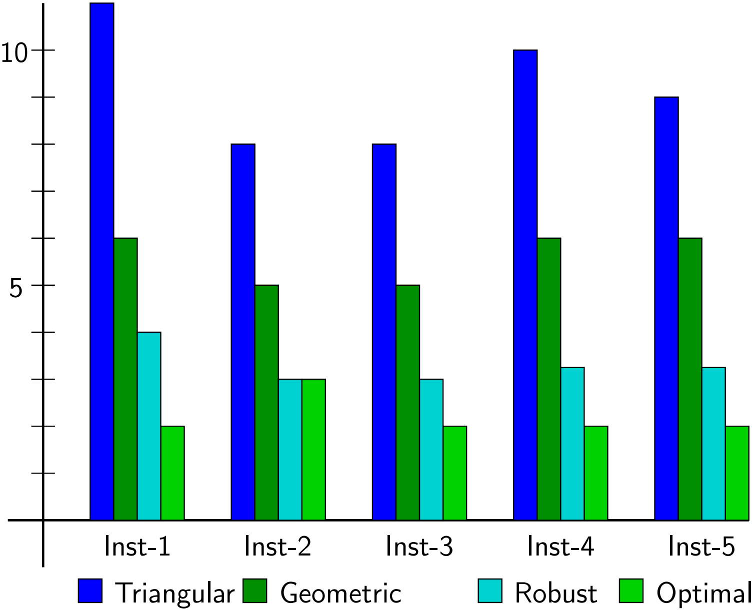
Find:

- Schedule with minimum number of sorting steps
- recoverable

Results:

- Generic algorithm
- $\mathbf{NP}$-hard
- Delay of j trains: in $\mathbf{P}$

Computational Results



Results

- k-Distance Recoverable Robustness
- Complexity, Cutting Planes
- Application to Train Classification

Summary

Results

- k-Distance Recoverable Robustness
- Complexity, Cutting Planes
- Application to Train Classification

Open Questions

- Compact Formulations
- Column Generation

Results

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Thank you for your attention!