Solving The Time Surplus Maximisation Bi-objective User Equilibrium Model of Traffic Assignment

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Outline

1. Conventional traffic assignment

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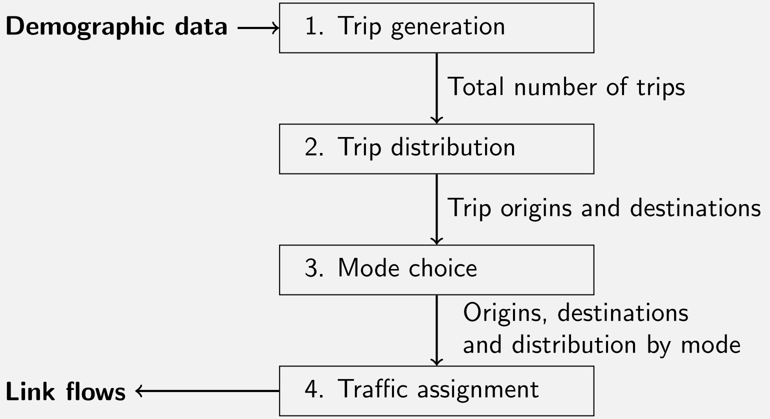
Transportation planning

Four-step process

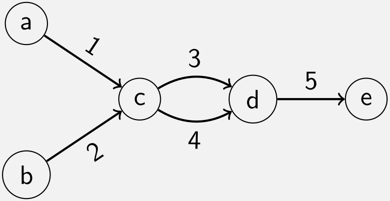


1. Conventional traffic assignment

Problem formulation

Input data

1. Transportation network $G(N, A)$, N – set of nodes, A – set of links



1. Conventional traffic assignment

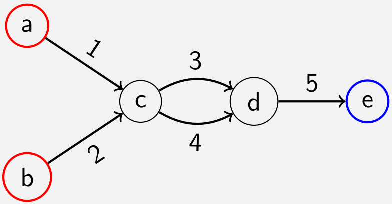
Problem formulation

Input data

1. Transportation network $G(N, A)$, N – set of nodes, A – set of links

2. Set of origin-destination (O-D) pairs Z and corresponding *demands* $D_p, \forall p \in Z$

Origin	Destination	D_p
a	e	10
b	e	20



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Origin	Destination	D_p
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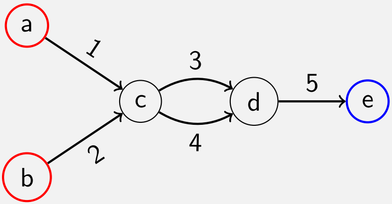
b	e	20

3. Link performance functions $c_a, \forall a \in A$



1. Conventional traffic assignment

Problem formulation

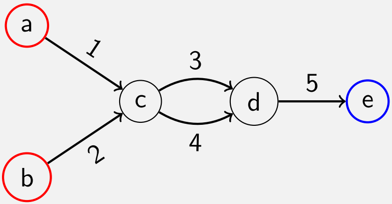
Output data

Link flow f_a – the number of vehicles per time unit travelling on link a

Path flow F_k – the number of vehicles per time unit travelling on path k

Path and link flows relation:

$f_1 = F_1 + F_2$
$f_2 = F_3 + F_4$
$f_3 = F_1 + F_3$
$f_4 = F_2 + F_4$
$f_5 = F_1 + F_2 + F_3 + F_4$



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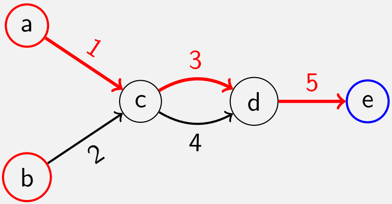
Flow on the first path: F_1



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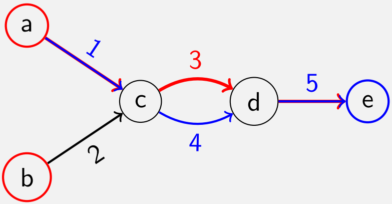
Flow on the first path: F_1

Flow on the second path: F_2



1. Conventional traffic assignment

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$f_3 = F_1 + F_3$
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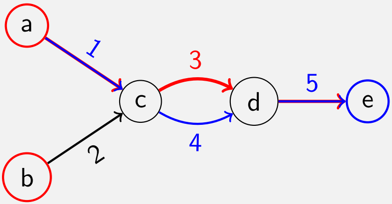
$f_5 = F_1 + F_2 + F_3 + F_4$

Flow on the first path: F_1

Flow on the second path: F_2

Link flow on link 1: $f_1 = F_1 + F_2$



1. Conventional traffic assignment

Wardrop's first principle

Wardrop, (1952)

The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route

User equilibrium conditions

$$F_k^* \left(C_k(\mathbf{F}^*) - U_p^*\right) = 0, \qquad \forall k \in K_p, \forall p \in Z, \tag{1}$$

$$C_k(\mathbf{F}^*) - U_p^* \geq 0, \qquad \forall k \in K_p, \forall p \in Z, \tag{2}$$

$$\sum_{k \in K_p} F_k^* - D_p = 0, \qquad \forall p \in Z, \tag{3}$$

$$F_k^* \geq 0, \qquad \forall k \in K_p, \forall p \in Z, \tag{4}$$

$$U_p^* \geq 0, \qquad \forall p \in Z. \tag{5}$$

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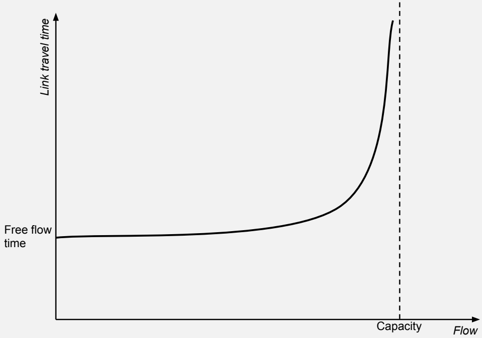
1. Conventional traffic assignment

Travel time modelling

Link performance function

Travel time of each link is usually defined as a function $c_a(\mathbf{f})$ (called *link performance function*) that is known for each link $a \in A$ and that might depend on the flow on all links $\mathbf{f} = (f_1, \ldots, f_m)$.



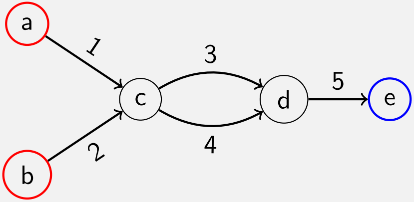
1. Conventional traffic assignment

Example (Sheffi, 1985)

Input data

Origin	Destination	D_p
a	e	2
b	e	3

Link	Performance function
1	$c_1 = 1$
2	$c_2 = 2$
3	$c_3 = 2 + f_3$
4	$c_4 = 1 + 2f_4$
5	$c_5 = 1$



1. Conventional traffic assignment

Example (Sheffi, 1985)

User equilibrium conditions

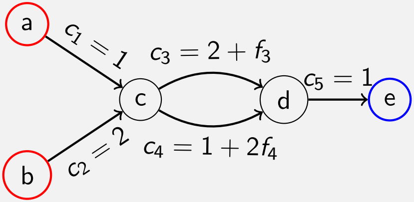
O-D pair a-e:
$C_1(\mathbf{F}) = C_2(\mathbf{F})$

O-D pair b-e:
$C_3(\mathbf{F}) = C_4(\mathbf{F})$

Path and link flows relation:

$f_1 = F_1 + F_2$
$f_2 = F_3 + F_4$
$f_3 = F_1 + F_3$
$f_4 = F_2 + F_4$
$f_5 = F_1 + F_2 + F_3 + F_4$

Origin	Destination	D_p
a	e	2
b	e	3



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1. Conventional traffic assignment

Example (Sheffi, 1985)

User equilibrium conditions

O-D pair a-e:

$C_1(\mathbf{F}) = C_2(\mathbf{F})$

$C_1(\mathbf{F}) = c_1(f_1) + c_3(f_3) + c_5(f_5)$

$C_2(\mathbf{F}) = c_1(f_1) + c_4(f_4) + c_5(f_5)$

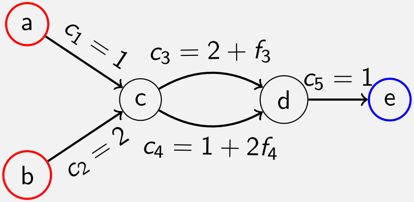
O-D pair b-e:

$C_3(\mathbf{F}) = C_4(\mathbf{F})$

$C_3(\mathbf{F}) = c_2(f_2) + c_3(f_3) + c_5(f_5)$

$C_4(\mathbf{F}) = c_2(f_2) + c_4(f_4) + c_5(f_5)$

Origin	Destination	D_p
a	e	2
b	e	3



1. Conventional traffic assignment

Example (Sheffi, 1985)

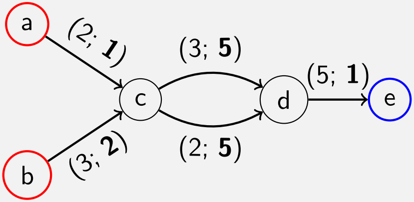
User equilibrium solution

$f_1 = 2$
$f_2 = 3$
$f_3 = 3$
$f_4 = 2$
$f_5 = 5$

O-D pair a-e:
$C_1(\mathbf{F}) = C_2(\mathbf{F}) = 7$

O-D pair b-e:
$C_3(\mathbf{F}) = C_4(\mathbf{F}) = 8$

Origin	Destination	D_p
a	e	2
b	e	3



1. Conventional traffic assignment

Assumptions

Assumptions

1. *Additivity* of the path cost functions: the cost of the path is just the sum of the costs of the links belonging to this path, i.e. $C_k(\mathbf{F}) = \sum_{a \in k} c_a(\mathbf{f}), \forall k \in K_p, \forall p \in Z$;

2. *Separability* of the link cost functions: the cost of the link depends only on the flow on this link, i.e $c_a(\mathbf{f}) = c_a(f_a), \forall a \in A$.

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1. Conventional traffic assignment

Mathematical program

Formulation (Sheffi, 1985)

$$
\begin{aligned}
&\min \sum_{a \in A} \int_0^{f_a} c_a(x)dx \\
&\sum_{k \in K_p} F_k = D_p, && \forall p \in Z, \\
&F_k \geq 0, && \forall k \in K_p, \forall p \in Z, \\
&f_a = \sum_{p \in Z} \sum_{k \in K_p} \delta_a^k F_k, && \forall a \in A.
\end{aligned}
\tag{6}
$$

2. Bi-objective traffic assignment

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2.1. Bi-objective user equilibrium

Definition (Wang et al. 2010)

Under **bi-objective user equilibrium (BUE) condition**, traffic arranges itself in such a way that no individual trip maker can improve either their toll or travel time or both without worsening the other component by unilaterally switching routes.

Notation

$C_k^{(1)}(\mathbf{F})$, $C_k^{(2)}(\mathbf{F})$ denote the objectives (for example, travel time and toll). The cost vector $\mathbf{C_k}(\mathbf{F}) = \left(C_k^{(1)}(\mathbf{F}), C_k^{(2)}(\mathbf{F})\right)$ is associated with each path $k \in K_p, \forall p \in Z$.

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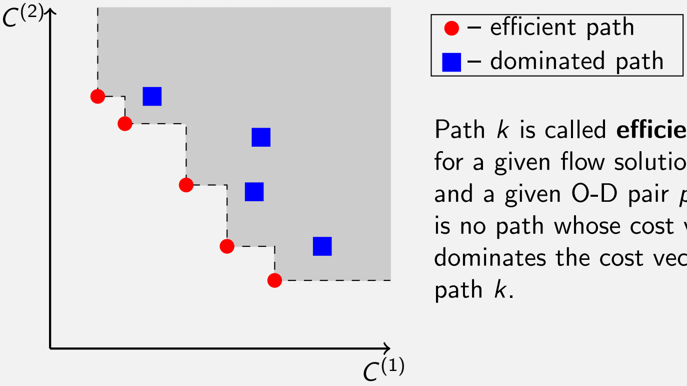
2. Bi-objective traffic assignment

2.1. Bi-objective user equilibrium

Efficient paths

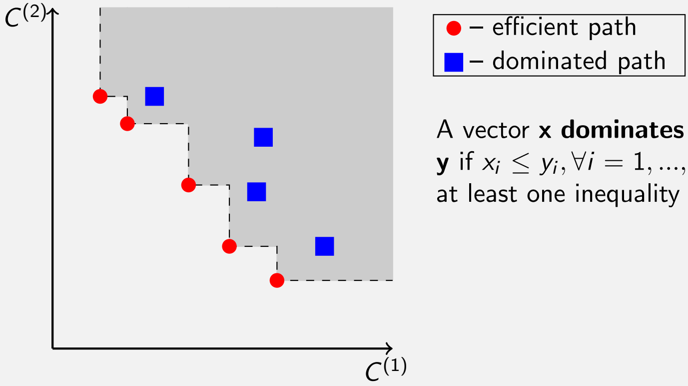


Path k is called **efficient** if for a given flow solution $\mathbf{F}$ and a given O-D pair p, there is no path whose cost vector dominates the cost vector of path k.

2. Bi-objective traffic assignment

2.1. Bi-objective user equilibrium

Efficient paths



A vector **x dominates** vector **y** if $x_i \leq y_i, \forall i = 1, ..., |\mathbf{x}|$ and at least one inequality is strict

2. Bi-objective traffic assignment

2.1. Bi-objective user equilibrium

Mathematical formulation (Raith et al., 2011)

A vector $\mathbf{F}^* \in \Omega$ satisfies BUE if for each O-D pair p the following statement holds:

$$\mathbf{C_k}(\mathbf{F}^*) \leq \mathbf{C_{k'}}(\mathbf{F}^*) \Rightarrow F^*_{k'} = 0, \quad \forall k \in K_p, \forall k' \in K_p, k' \neq k, \qquad (7)$$

i.e. the flow is positive only on efficient paths, the paths that carry zero flow are either dominated or efficient.

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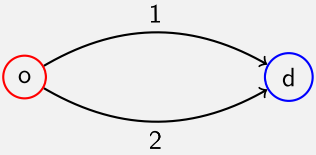
2. Bi-objective traffic assignment

2.2. Example

Input data

Origin	Destination	D_p
o	d	5

Link	Performance function	Toll
1	$c_1 = f_1 + 1$	$\tau_1 = 1$
2	$c_2 = 2f_2 + 1$	$\tau_2 = 2$



2. Bi-objective traffic assignment

2.2. Example

BUE conditions

Since:

$C_1^{(2)} < C_2^{(2)}$

$\tau_1 < \tau_2$

then:

$C_1^{(1)}(\mathbf{F}) > C_2^{(1)}(\mathbf{F})$

$c_1(f_1) > c_2(f_2)$

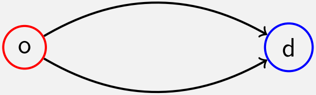
Path and link flows relation:

$f_1 = F_1$

$f_2 = F_2$

Origin	Destination	D_p
o	d	5

$c_1 = f_1 + 1; \tau_1 = 1$



$c_2 = 2f_2 + 1; \tau_2 = 2$

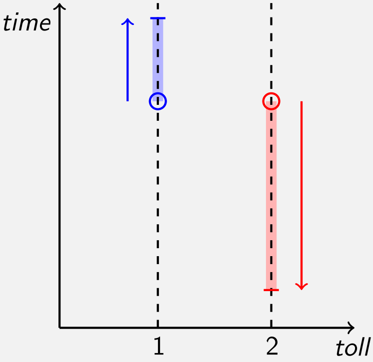
2. Bi-objective traffic assignment

2.2. Example

BUE solution space

$$\begin{cases} f_1 > 3\frac{1}{3} \\ f_1 \leq 5 \\ f_2 = 5 - f_1 \end{cases}$$

Infinitely many solutions satisfy the BUE condition



2. Bi-objective traffic assignment

2.3. Aggregation of objectives

Aggregation of objectives

Aggregation of objectives

Linear aggregation of objectives:

$\alpha_1 C_k^{(1)}(\mathbf{F}) + \alpha_2 C_k^{(2)}(\mathbf{F})$

→ Only supported efficient solutions

Non-linear aggregation of objectives:

$C_k^{(1)}(\mathbf{F}) + v(C_k^{(2)}(\mathbf{F}))$

→ Possible to find any solution satisfying the BUE condition

3. Time surplus model

3. Time surplus model

3.1. Definition

Definition (Wang and Ehrgott, 2011)

Time surplus is defined as the maximum time a user is willing to spend during the travel minus the actual time spent:

$$v(C_k^{(2)}) - C_k^{(1)}(\mathbf{F}) \tag{8}$$

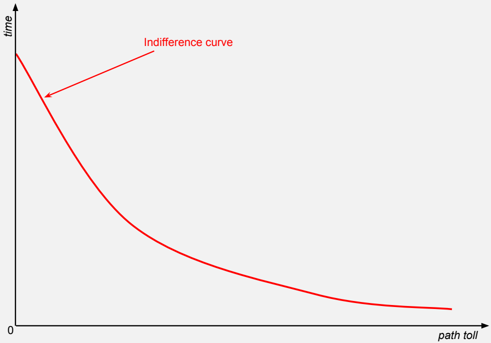
The maximum time a user is willing to spend is modelled as an **indifference curve** – a linear or non-linear function that depends on the path toll.

Under **the TSMaxBUE condition**, traffic arranges itself in such a way that no individual trip maker can improve his/her time surplus by unilaterally switching routes, or alternatively, all individuals are travelling on the path with the highest time surplus value among all the efficient paths between each origin-destination pair.

3. Time surplus model

3.1. Definition

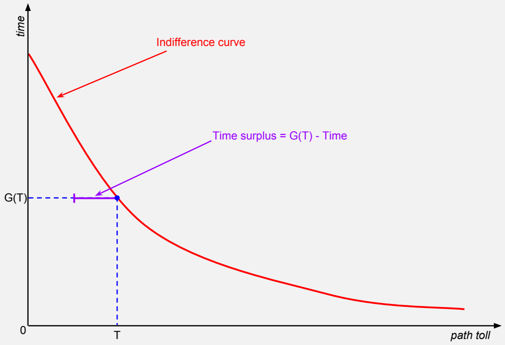
Indifference curve



3. Time surplus model

3.1. Definition

Indifference curve



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3. Time surplus model

3.1. Definition

User classes

- Under the assumption that all users have the same indifference curve, we obtain TSMaxBUE with **one user class**;
- When all drivers are divided into classes with different indifference curves, we obtain TSMaxBUE with **multiple user classes**.

3. Time surplus model

3.2. Single user case

Assumptions

- All link cost functions are separable;
- Path travel time is additive;
- Path toll does not depend on the flow and is additive;
- Indifference curves $G_p(T_k)$ are non-negative continuous and strictly decreasing functions of the path toll T_k.

3. Time surplus model

3.2. Single user case

Single user case

Mathematical formulation (Larsson et al., 2002):

$$
\begin{aligned}
&\min \sum_{a \in A} \int_0^{f_a} c_a(x)dx + \sum_{p \in Z} \sum_{k \in K_p} F_k \overline{G_p}(T_k) && \\
&\sum_{k \in K_p} F_k = D_p, && \forall p \in Z, \\
&F_k \geq 0, && \forall k \in K_p, \forall p \in Z, \\
&f_a = \sum_{p \in Z} \sum_{k \in K_p} \delta_a^k F_k, && \forall a \in A.
\end{aligned} \tag{9}
$$

where $\overline{G_p}(T_k) = G_p(0) - G_p(T_k)$.

3. Time surplus model

3.2. Single user case

Non-additive shortest path

Cost of path k:

$$C_k(\mathbf{f}) = \sum_{a \in k} c_k(f_k) + \overline{G_p}(T_k) \tag{10}$$

is **non-additive**.

Under the assumption that $\overline{G_p}(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+, \forall p \in Z$ is strictly increasing, the shortest path in terms of the generalised cost (10) is one of the **efficient paths**.

Algorithm

Path equilibration algorithm with bi-objective shortest path algorithm used to solve non-additive subproblem can be applied.

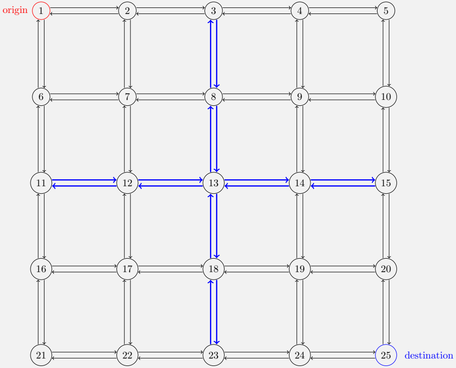
3. Time surplus model

3.3. Small instance study

Instance

Solution method:
path equilibration algorithm

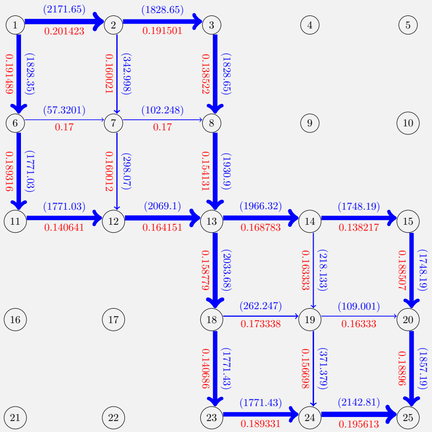
Bi-objective shortest path algorithm:
label setting



3. Time surplus model

3.3. Small instance study

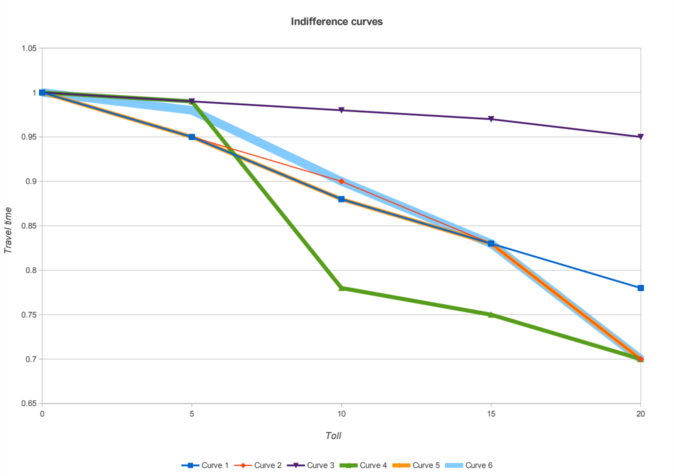
Conventional user equilibrium solution



3. Time surplus model

3.3. Small instance study

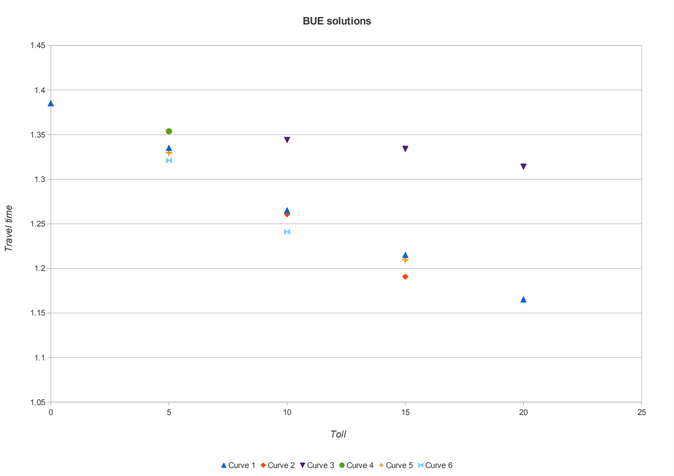
Indifference curves



3. Time surplus model

3.3. Small instance study

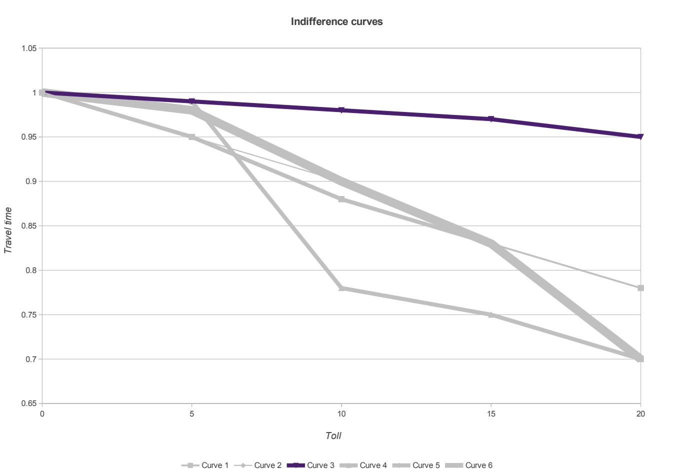
Images of the solutions corresponding to indifference curves



3. Time surplus model

3.3. Small instance study

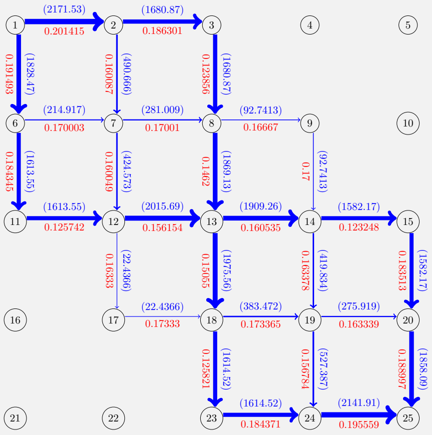
Indifference curves



3. Time surplus model

3.3. Small instance study

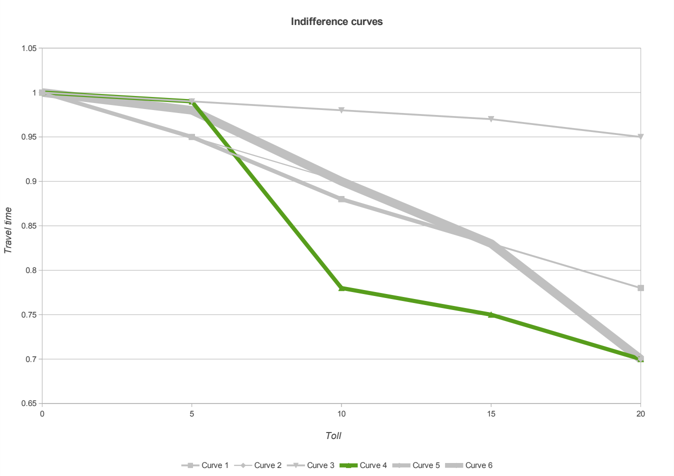
Solution corresponding to Curve 3



3. Time surplus model

3.3. Small instance study

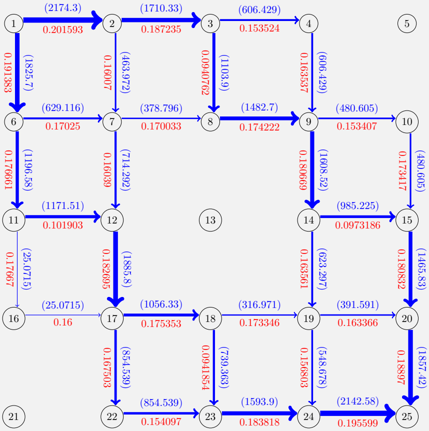
Indifference curves



3. Time surplus model

3.3. Small instance study

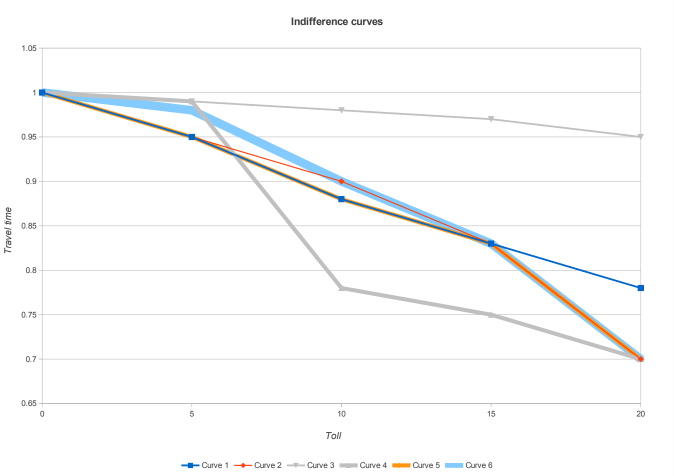
Solution corresponding to Curve 4



3. Time surplus model

3.3. Small instance study

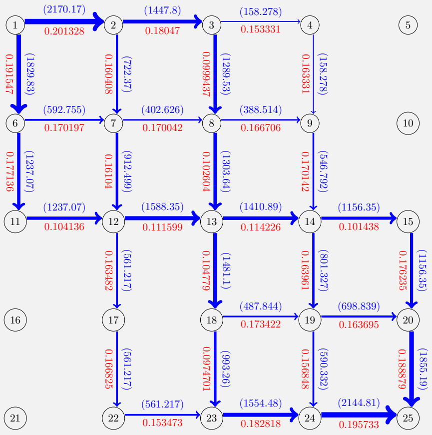
Indifference curves



3. Time surplus model

3.3. Small instance study

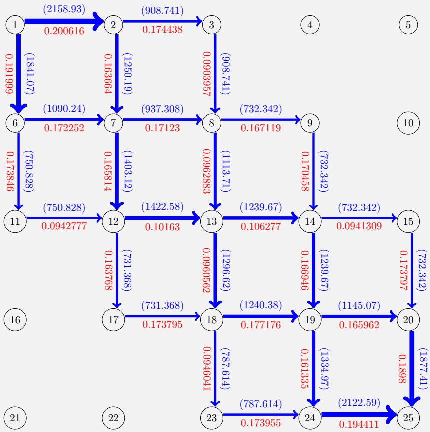
Solution corresponding to Curve 1



3. Time surplus model

3.3. Small instance study

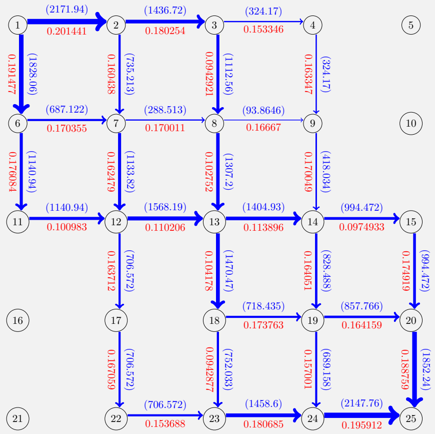
Solution corresponding to Curve 2



3. Time surplus model

3.3. Small instance study

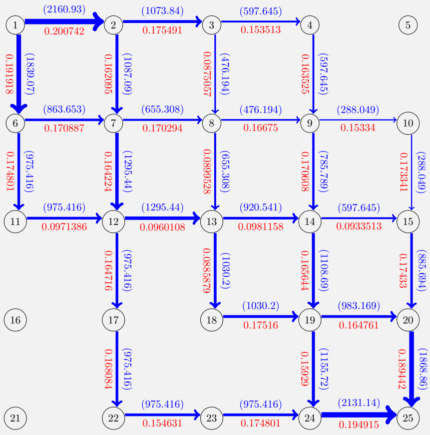
Solution corresponding to Curve 5



3. Time surplus model

3.3. Small instance study

Solution corresponding to Curve 6



3. Time surplus model

3.4. Computational study on bigger instances

Instances `http://www.bgu.ac.il/~bargera/tntp/`, tolls generated using marginal costs and then scaled

Instance name	Nodes	Arcs	Zones	O-D pairs
Sioux-Falls	24	76	24	528
Anaheim	416	914	38	1406
Barcelona	1020	2522	110	7922

Environment

- Programming language: C++, compiler: g++ 4.6.3;
- OS: Ubuntu Release 12.04 64-bit, Kernel Linux 3.2.0-24-generic;
- CPU: Intel Core i5-2500 CPU, 4 Core, 3.30GHz;
- RAM: 7.7 GB.

3. Time surplus model

3.4. Computational study on bigger instances

Algorithms

- **Path equilibration (PE)**: shift flow from the longest path to the shortest path (Dafermos and Sparrow, 1969);
- **Gradient projection (GP)**: shift flow from non-shortest paths to the current shortest path (Jayakrishnan et al., 1994);
- **Projected gradient (PG)**: flow is shifted from the paths that have cost greater than the current average path cost to the paths that have cost less than the average value (Florian et al., 2009).

Convergence measure

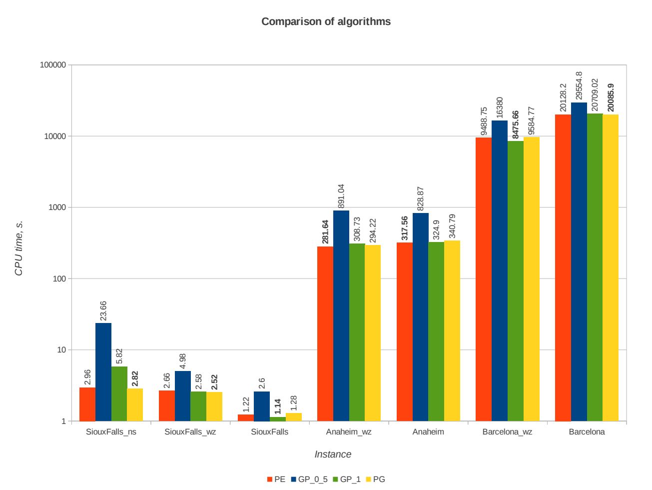
Convergence rate:

$$convRate = \max_{p \in Z, k \in K_p^+} C_k^{max}(\mathbf{f}) - C_k^{min}(\mathbf{f}). \qquad (11)$$

3. Time surplus model

3.4. Computational study on bigger instances

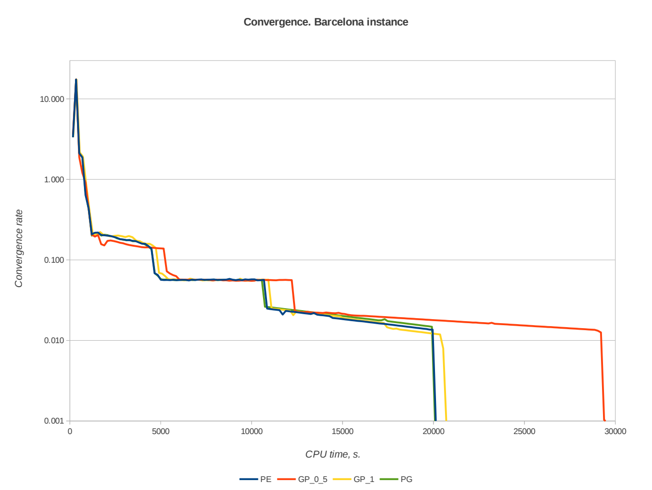
Results



3. Time surplus model

3.4. Computational study on bigger instances

Convergence



4. Conclusion and future work

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Conclusion

- The layout of the indifference curves is similar to the layout of the corresponding images of the solutions in objective space.
- By changing the ratio between possible values of travel time corresponding to different values of toll, it is possible to generate different flow patterns.
- The TSMaxBUE model provides an efficient and flexible tool able to solve BUE problem.

Future work

- Implement more efficient bi-objective shortest path algorithms;
- Extend TSMaxBUE model to multiple user case;
- Consider a flow-dependent second objective.

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Solving The Time Surplus Maximisation Bi-objective User Equilibrium Model of Traffic Assignment

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