FP7-PEOPLE-2009-IRSES:
Project ID 246647

Optimization and its Applications in Learning and Industry (OptALI)

IRSES

Ongoing Deliverable D1.2

Description of Research Seminar: Variants of the Shortest Path Problem

Start date of the Workpackage:	December 2010
Duration:	48 months
Due date of deliverable:	November 2014
Actual submission date:	January 2013
Participants:	UGOE UNIKL DTU UOA UC
Author of deliverable:	L. Turner (turner@mathematik.uni-kl.de)

Research Seminar

offered by Lara Turner (ES-UNIKL)

in February 2012,

in Auckland, New Zealand

Subject: Variants of the Shortest Path Problem

Problem: The sum shortest path problem in which the (s,t)-paths P of a given digraph $G=(V,E)$ with $n:=|V|$ and $m:=|E|$ are compared with respect to the sum of their edge costs is one of the best known problems in combinatorial optimization. Variations of this problem with different objective functions are the bottleneck, balanced, minimum deviation and algebraic sum, k-sum and k-max shortest path problems on the one hand as well as the (k_1,k_2)-max and (k_1,k_2)-balanced, (k_1+k_2)-trimmed-mean and (k_1+k_2)-anti-trimmed-mean shortest path problems on the other hand. While efficient algorithms for the first class of problems exist, shortest path problems with (k_1,k_2)-max, (k_1,k_2)-balanced or trimmed-mean objectives have not yet been treated in literature. In this research seminar, we propose a general model for solving these problems in strongly polynomial time. It is based on the solution of resource constrained shortest path problems with equality constraints.

Main Results: The objective functions mentioned above arise as special cases of the more general universal shortest path problem (Univ-SPP(l)) which is of the sum type and associates multiplicative weights $\lambda_i^l \in \mathbb{R}$ with the ordered edge costs $c_{(i)}(P)$ of an elementary path $P \in \mathcal{P}_{st}$:

$$\min_{P\in\mathcal{P}_{st}:\ l(P)=l} \ \sum_{i=1}^{l} \lambda_i^l \cdot c_{(i)}(P)$$

Here, $c_{(i)}(P)$, $i=1,\ldots,l$, denotes the i^{th}-largest edge cost in path P and $l=l(P)\in\{1,\ldots,n-1\}$ is the length of path P. This is the sequential definition of the universal shortest path problem, and, by choosing appropriate weight coefficients, classical and new objective functions can be modeled by Univ-SPP(l). For instance, we obtain the sum objective function by setting $\lambda_i^l = 1$ for all $i = 1, \ldots, l$ and $l = 1, \ldots, n-1$. The $(k_1 + k_2)$-trimmed-mean objective function on which we focus in the research seminar is modeled by setting $\lambda_{k_1+1}^l, \ldots, \lambda_{l-k_2}^l = 1$ and $\lambda_i^l = 0$ else for all $l \geq k_1 + k_2$. For an (s,t)-path P of length $l(P) \geq k_1 + k_2$, it ignores the k_1 largest and k_2 smallest edge costs and adds the costs of the remaining edges. Although Univ-SPP(l) is, in general, strongly NP-hard, we can show that (k_1+k_2)-trimmed-mean shortest path problems (and, similarly, (k_1, k_2)-max, (k_1, k_2)-balanced and $(k_1 + k_2)$-anti-trimmed-mean shortest path problems) can be solved in strongly polynomial time. Sorting the edges $e \in E$ by non-increasing costs such that $c(e_1) \geq \ldots \geq c(e_m)$, the idea is to iteratively fix an edge $e_{j_{k_1}} \in E$ as the k_1^{st}-largest cost edge of the elementary paths $P \in \mathcal{P}_{st}$, and, thus, to partition the set $\mathcal{P}_{st}$ into sets $\mathcal{P}_{st}(e_{j_{k_1}})$ containing all elementary paths $P \in \mathcal{P}_{st}$ with the edge $e_{j_{k_1}} \in E$ as the k_1^{st}-largest cost edge. This can be done by introducing binary weights and yields resource constrained shortest path problems with equality constraints. Since the number of resources is fixed and the resource limits are bounded in the input size, these problems can be solved in $\mathcal{O}(n^2m)$ time on acyclic digraphs $G = (V, E)$ by applying dynamic programming. Together with an appropriate definition of edge costs $c_{j_{k_1},j_{k_2}}(e_i)$, $e_i \in E$, and results from k-sum optimization (see Punnen and Aneja, 1996), we prove that the (k_1+k_2)-trimmed-mean shortest path can be found among these resource constrained shortest paths. More precisely, we can show that all paths $P \in \mathcal{P}_{st}(e_{j_{k_1}}, e_{j_{k_2}})$ with edge $e_{j_{k_1}}$ as k_1^{st}-largest cost edge and edge $e_{j_{k_2}}$ as k_2^{nd}-smallest cost edge have a trimmed-mean objective function value which is larger or equal than the trimmed-mean objective function value of a resource constrained shortest path with respect to costs $c_{j_{k_1},j_{k_2}}(e_i)$ and set $\mathcal{P}_{st}(e_{j_{k_1}})$. Therefore, at most m^2 resource constrained shortest path problems need to be solved and we obtain an $\mathcal{O}(n^2m^3)$ time algorithm for the $(k_1 + k_2)$-trimmed-mean shortest path problem on acyclic digraphs $G = (V, E)$. For general digraphs, the time complexity is of order $\mathcal{O}(n^3m^3)$.

Participants: Students and researchers from UOA. Early stage and experienced researchers from UGOE, UNIKL and UOA.

Publication: Turner, L., Variants of the Shortest Path Problem, Algorithmic Operations Research, 6(2): 91-104. (2011)