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Description of Research Seminar: A polynomial time approach for the multiple objective minimum spanning tree problem

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Participants:	UGOE UNIKL DTU UOA UC
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Research Seminar

offered by Florian Seipp (UNIKL)

in March 2012,

in Auckland, New Zealand

Subject: A polynomial time approach for the multiple objective minimum spanning tree problem

Problem: The minimum spanning tree problem is a well-studied problem in combinatorial optimization: the task of identifying a spanning tree of a given connected graph which has minimal cost with respect to the sum objective. Whereas the single objective version of this problem can be solved in polynomial time by greedy algorithms, the *multiple objective minimum spanning tree problem (MOMST)* is known to be intractable and $\mathcal{NP}$-hard. These negative results already hold for the case of two objective functions. Even worse, the number of both supported and unsupported nondominated points may be exponentially large in the number of vertices of the underlying graph. This immediately raises the question about the number of nondominated extreme points of the MOMST problem; these are sufficient to describe the nondominated frontier.

Main Results: Supported nondominated points of a multiple objective combinatorial optimization problem are the ones that can be obtained by solving a weighted sum scalarization with some positive (in all components) scalarization vector. The nondominated extreme points constitute special cases, since each of them is the unique objective vector of the optimal solution(s) of some weighted sum problem.

For the MOMST problem, a weighted sum problem is nothing but an ordinary (i.e. single criterion) MST problem in which the originally vector-valued edge costs are weighted. Hence, these problems can be solved by the well-known greedy algorithms. The outcome of a greedy algorithm only depends on the order of the edges, i.e., the sorting with respect to the weighted edge costs, not on the actual resulting cost values. Thus, the number of nondominated extreme points of MOMST is bounded from above by the number of possible different sortings of the edges in dependence of the scalarization vector.

Based on the theory of *arrangements of hyperplanes*, we show that this number does not exceed a polynomial in the size of the input data. For each pair of distinct edges, we introduce a separating hyperplane in the weight space representing the set of all relevant weighting vectors of scalarizations for which the weighted edge costs of these two edges coincide. These hyperplanes subdivide the weight space into regions that correspond to those weighting vectors yielding a unique order of the edges. It is a well-known fact that the number of resulting regions is bounded by a polynomial in the number of separating hyperplanes.

The result immediately implies that the computation of the nondominated frontier can be accomplished by solving polynomially many weighted sum problems. We present a solution approach which demonstrates how this can be achieved algorithmically for the case of two objective functions. Finally, we give an outline for the generalization of the method to an arbitrary number of objectives.

Participants: Students from UOA, early stage and experienced researchers from UOA, UGOE, and UNIKL.

Publication: —