Technische Universität Kaiserslautern

The Empty Freight Car Distribution Problem

in collaboration with DB SCHENKER

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OPT Optimization Research Group

Problem Definition

Situation:

- Loading and unloading of freight cars in different locations
- Unloading freight cars → supply
- Loading freight cars → demand

Problem:

At some places: # supply ≠ # demand

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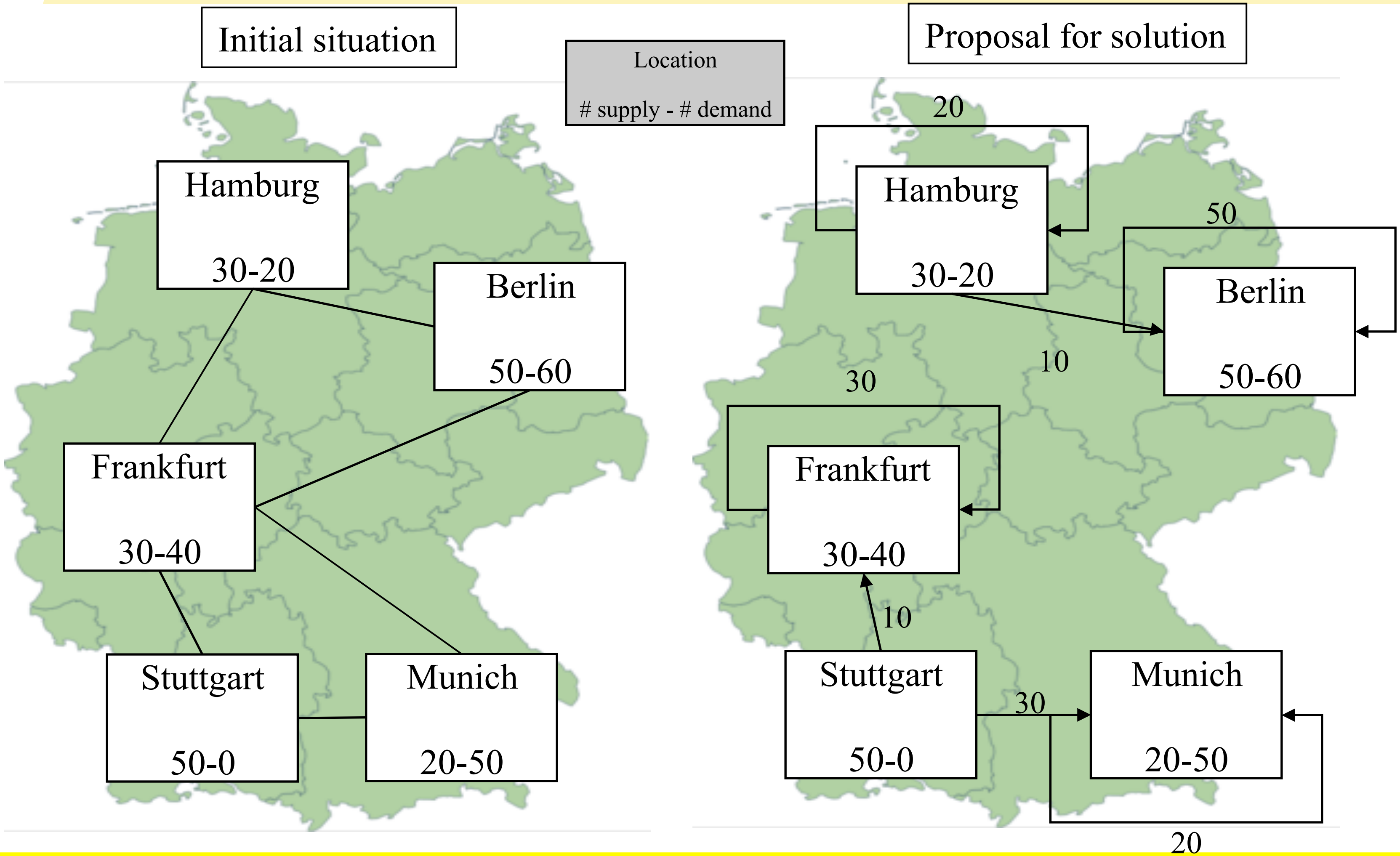
Empty freight car movements

Aim:

Empty Freight Car Distribution with minimum transport costs

Example


Initial situation
Location
supply - # demand
Proposal for solution
Hamburg
30-20
Berlin
50-60
Frankfurt
30-40
Stuttgart
50-0
Munich
20-50
20
50
10
30
10
30
20

Supplies / Demands / Connections

Supply

- Place of supply
- Number of cars
- Disposal time

Demand

- Place of demand
- Number of cars
- Required time

Suitable train connection

(Parts of) supply can satisfy demand (in parts)

Initial Network Model I

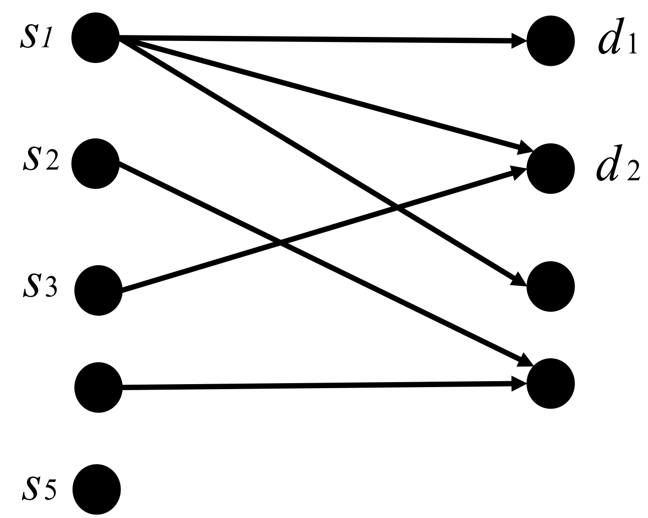
Components:

(1) Supply-Level

(2) Demand-Level

(3) Connection-Level

G=(V,R) with
- V= {supply set S, demand set D}
- R={(s,d) ∈ S × D: if s can satisfy d}



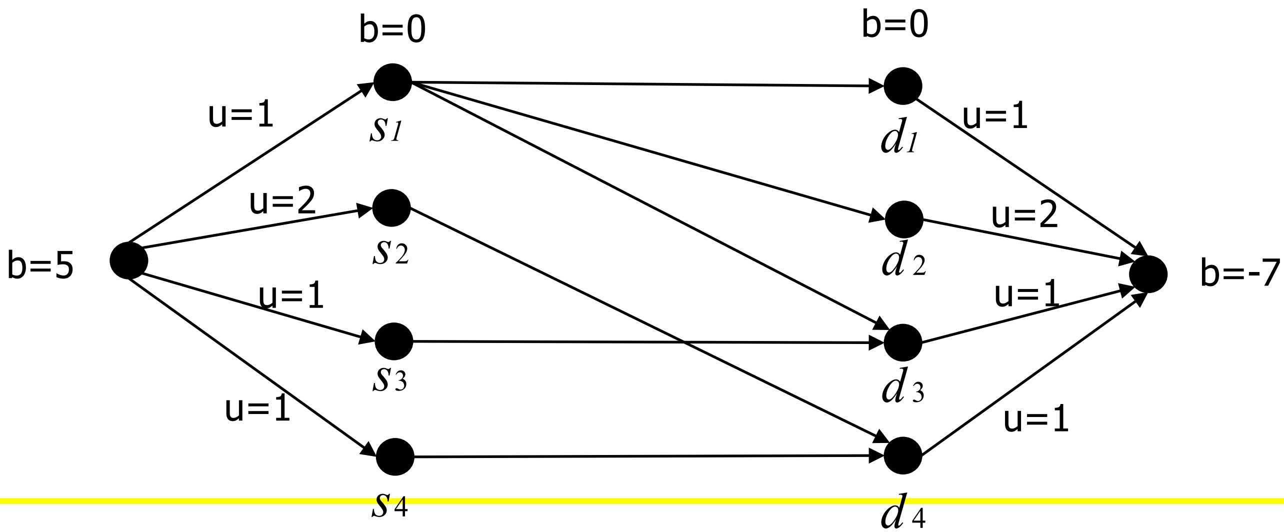
Initial Network Model II

„How can we model how much cars a supply / demand contains of?"

Extension of the Network Model

- Add a global sink / source, edges and capacities u
- Add a excess funktion b: V $\longrightarrow \mathbb{Z}$

i	s_i	d_i
1	1	1
2	2	2
3	1	3
4	1	1



Algorithmic Approach I

Consider the distribution problem as a network flow problem

- Existence of efficient algorithms
- Integer Solution when integer input data
- Taking advantage of special network structures

Modelling generates a minimum cost flow problem:

$$\min \sum_{(i,j)\in R} c_{i,j} \cdot x_{i,j}$$

$$\sum_{i:(i,j)\in R} x_{i,j} - \sum_{i:(j,i)\in R} x_{j,i} = b(j) \text{ for all } j \in A \cup B \quad (*)$$

$$0 \le x_{i,j} \le u_{i,j} \text{ for all } (i,j) \in R \quad (**)$$

$(*)$ mass balance contraints
$(**)$ capacity constraints

Algorithmic Approach II

Successive-Shortest-Path Algorithm (SSP)

- Conceptual easy approach
- Pseudo-polynomial running time
- Transformation into weak polynomial algorithms using scaling techniques

Procedure:

As long as there exists a path between unbalanced source and sink **do:**

(1) Send flow from source to sink along a shortest path in the residual network

(2) Update: network

Algorithmic Approach III

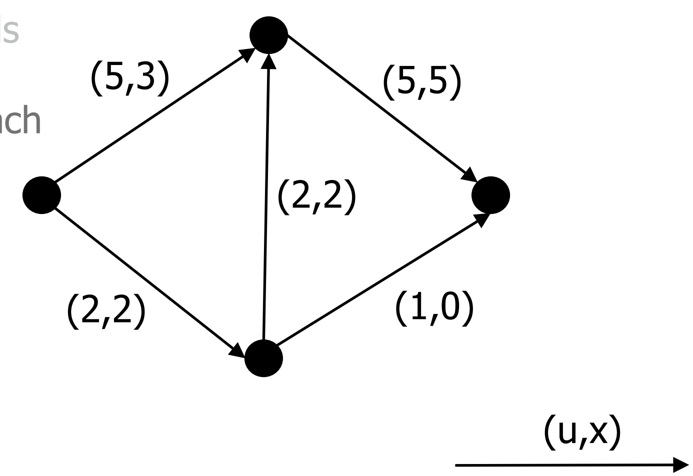
Procedure:

As long as there exists a path between unbalanced source and sink **do:**

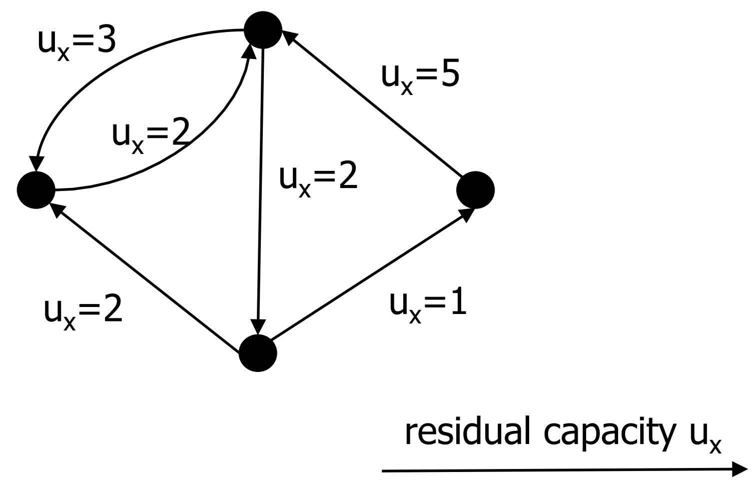
(1) Send flow from source to sink along a shortest path in the residual network

(2) Update: network

Network



Residual Network



Algorithmic Approach IV

Reduced-costs optimality condition

A flow x is optimal if and only if there exist a node potential $p: V \to \mathbb{R}$ with $c^p(r) := c(r) + p(u) - p(v) \geq 0$ for all $r=(u,v) \in G_x$.

Transforms **infeasible, super-optimal** solution into **feasible optimal** solution

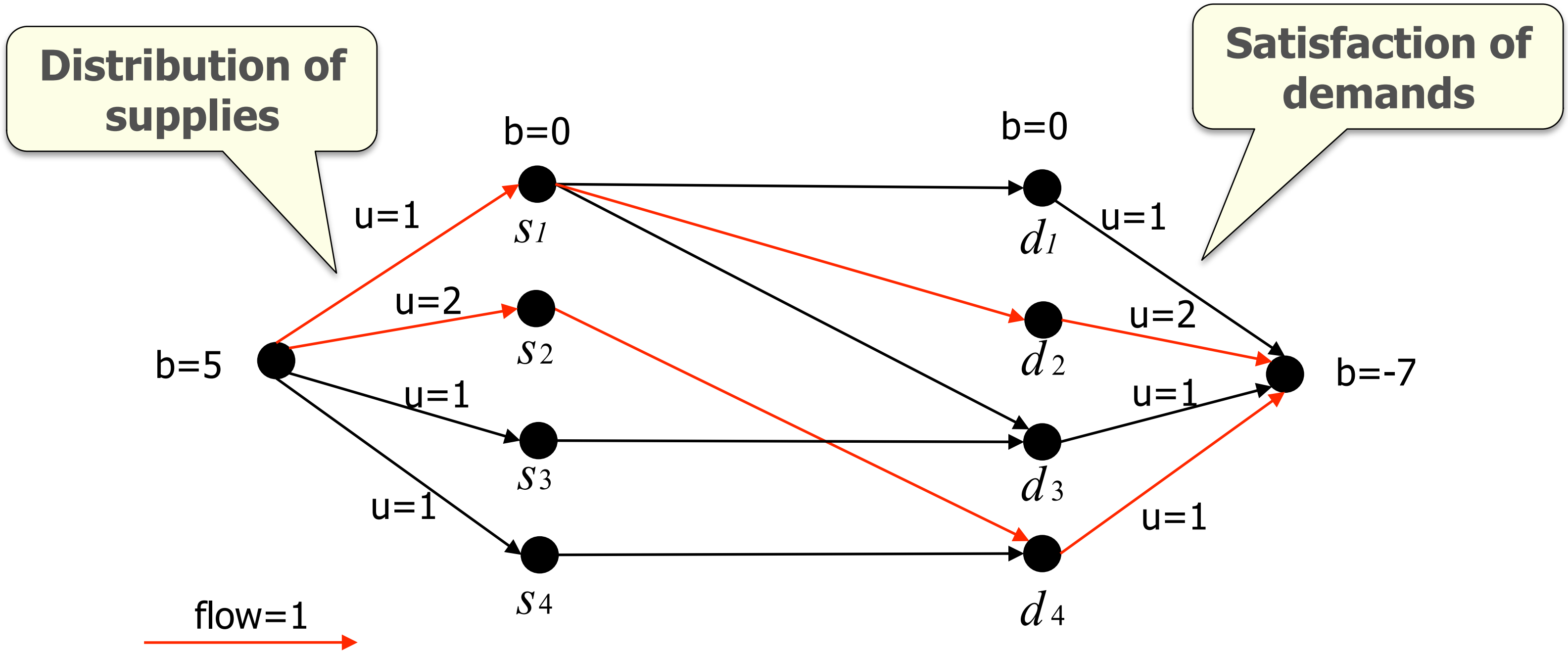
Shortest path computation w.r.t. positive reduced costs…Why?

- Shortest path computation with arbitrary costs: $O(nm) \subseteq O(n^3)$
- Shortest path computation with non-negative costs: $O(n \log n + m)$

Interpretation

„How can wie transform the min cost flow into a feasible distribution?"

Every unit of flow in connection-level corresponds to a car movement



The Reality

- European Distribution → **Huge input set**
- Car Types
- Substitution
- Priorization of demands
- Warmstarts
- …

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Continutity of the freight car distribution process

Car Types and Substitution

Background: Different car types

Given: different car types

„**One** car of type A can substitute **two** car of type B"

Use multiplier $\mu : R \rightarrow]0, \infty[$ on the edges

Generalized Min Cost Flow Problem

„**One** car of type A can substitute **one** car of type B"

Restriction of the connection- level

$$\min \sum_{(i,j)\in R} c_{i,j} \cdot x_{i,j}$$

$$\sum_{i:(i,j)\in R} \mu_{i,j} x_{i,j} - \sum_{i:(j,i)\in R} x_{j,i} = b(j) \text{ for all } j \in A \cup B$$

$$0 \leq x_{i,j} \leq u_{i,j} \text{ for all } (i,j) \in R$$

$$0 \leq \mu_{i,j} < \infty \text{ for all } (i,j) \in R$$

Substitution

Theorem (Garey et al):

The generalized flow problem is NP-hard to solve.

- Remains NP-hard if we just allow multiplier 1 and 2.
- Optimal solutions with fractional values can occur
- Existence of good heuristics

But in series-parallel graphs, we can solve it in

- Continuous Case → Polynomial time
- Integral Case → Pseudo-polynomial time

Further Requirements

- European Distribution → **Huge input set**
- Car Types → **Restriction of the connection- level**
- Substitution → **Generalized Flow Problem**
- Priorization of demands
- Warmstarts
- …

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Continuous freight car distribution process

Priorization of Demands

Background: Lack of empty freight cars

Weak Priorization

- No guarantee of satisfaction
- Via cost terms

Strong Priorization

- Guarantee of as much satisfaction as possible
- Not via cost terms

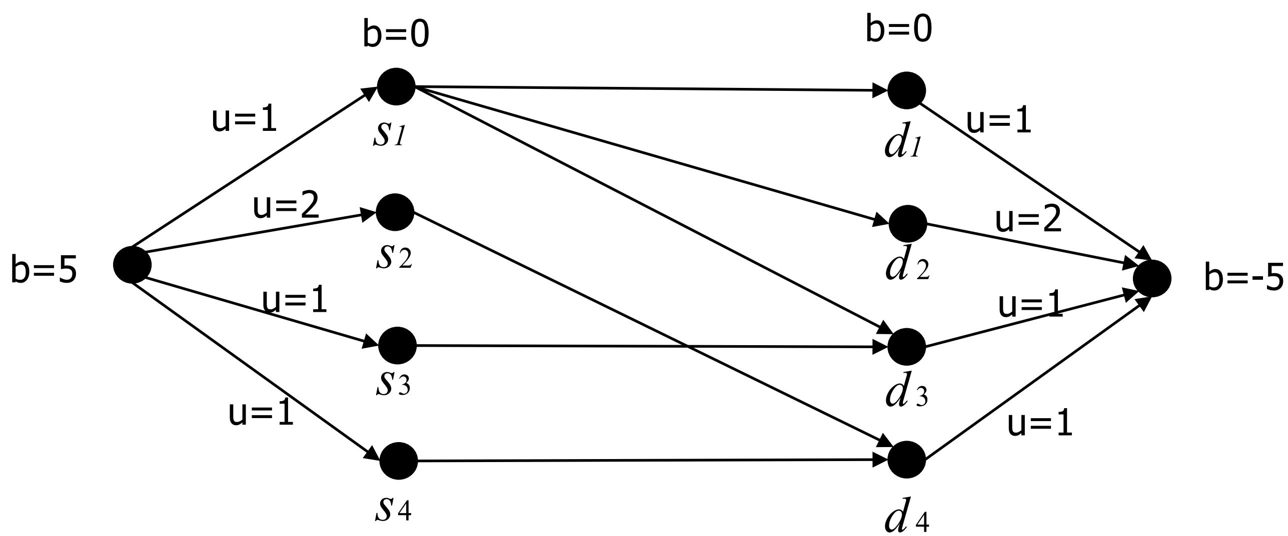
Modified network modell and algorithmic approach

Priorization of Demands: Network Modell

Idea: use two sinks

i	s_i	d_i
1	1	1
2	2	2
3	1	1
4	1	1

i	s_i	d_i	Priority of demand
1	1	1	strong
2	2	2	weak
3	1	1	weak
4	1	1	strong



Priorization of Demands: Network Modell

Idea: use two sinks

i	s_i	d_i
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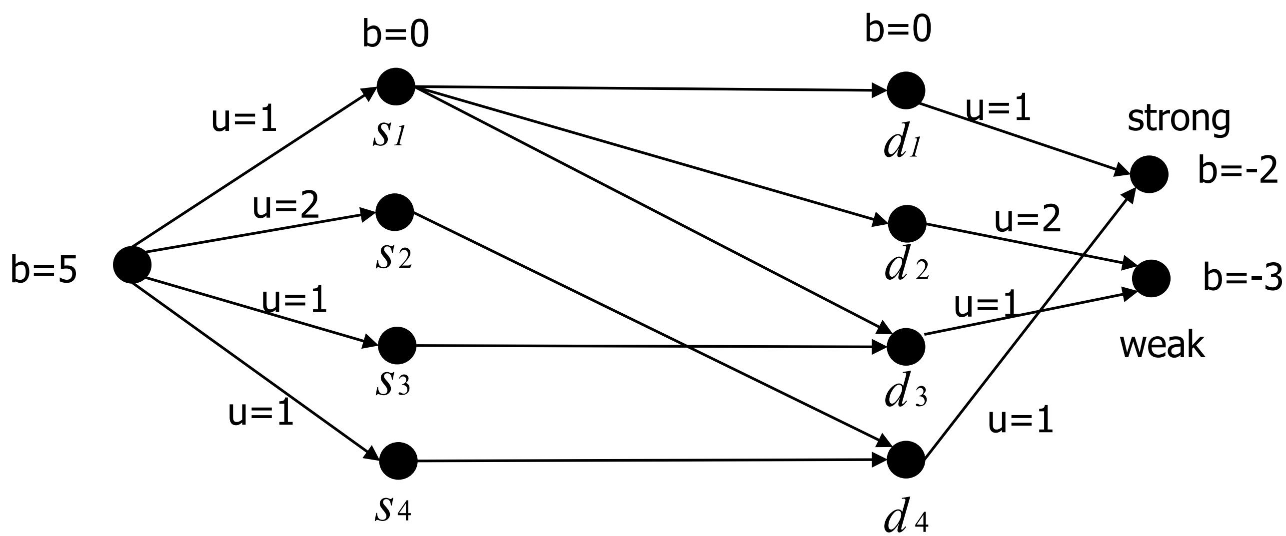
i	s_i	d_i	Priority of demand
1	1	1	strong
2	2	2	weak
3	1	1	weak
4	1	1	strong



Priorization of Demands: Algorithmic Approach

Procedure:

Strong Priorization

As long as there exists a path between unbalanced source and sink „strong" **do:**

(1) Send flow from source to sink „strong" along a shortest path in the residual network

(2) Update: network

THEN…

As long as there exists a path between unbalanced source and sink „weak" **do:**

(1) Send flow from source to sink „weak" along a shortest path in the residual network

(2) Update: network

Weak Priorization

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Further Requirements

- European Distribution → **Huge input set**
- Car Types → **Restriction of the connection- level**
- Substitution → **Generalized Flow Problem**
- Priorization of demands → **Modified SSP**
- Warmstarts → **Techniques from SSP**
- …

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Continutity of the freight car distribution process