Line Pool Generation with LinTim

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Overview

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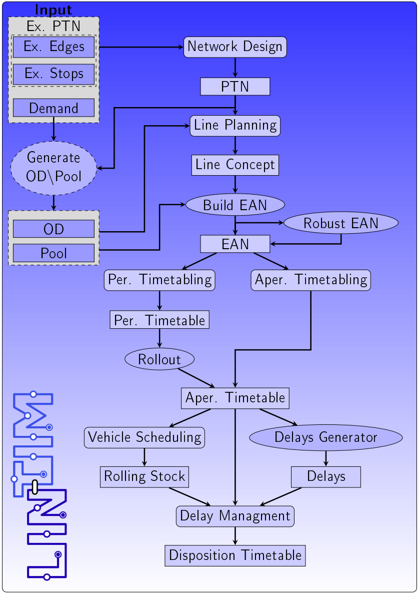
- Motivation and Differentiation from Line Planning
- Formalization
- Algorithm
- Results



Number of Paths in a Graph

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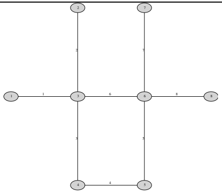	8	50	$<$ 1s

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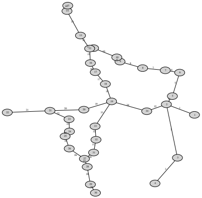	34	1636	2s

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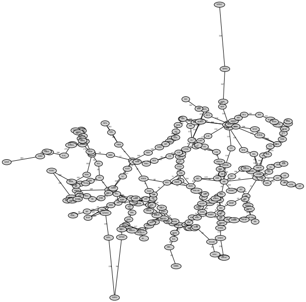	250	$>$ 1Mio	$>$ 100h

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Question: Do all lines have to be computed beforehand?

Different approaches

- Generate reasonable lines at the time of line planning ([BGP05, BGP07])

  Pros
  - Potentially all lines possible

  Cons
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  - Increasing computation time

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- Generate set of promising lines as input for the line planning ([Bus98, ZCD96, CDZ98])
  - Pros
    - Reduced complexity of line planning
    - Decreased computation time
    - Precise control of "promising"
    - Use of external knowledge
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Feasibility of the line concept

A line concept is feasible, if the frequency constraints

$$f_e^{min} \leq \sum_{l \in \mathcal{L}: e \in l} f_l \leq f_e^{max} \quad (1)$$

for all edges are not violated.

LP-Cost

Given costs $cost_l$ $(= \sum_{e \in l} c_e + c^{fix})$ for each line, minimize $\sum_{l \in \mathcal{L}} f_l cost_l$ such that (1) is fulfilled and $f_l \in \mathbb{N}_0$ for all $l \in \mathcal{L}$.

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LP-Direct

Maximize the number of direct travelers such that (1) is fulfilled and $f_l \in \mathbb{N}_0$ for all $l \in \mathcal{L}$.

Line Pool Generation Problem

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Problem: LPool

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\text{s.d.} \quad & \sum_{l \in \mathcal{L}^0 : e \in l} f_l \geq f_e^{min} \quad \forall e \in E \\
& \sum_{l \in \mathcal{L}^0 : e \in l} f_l \leq f_e^{max} \quad \forall e \in E \\
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What is a good line pool?

- Size
- Low costs
- Many direct travelers

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1. While there exists no feasible line concept
2. Compute a minimal spanning tree (MST) based on
   - first loop: shortest paths of passengers
   - other loops: edges, which prevent feasibility
3. Compute terminals as stations with
   - first loop: high node degree
   - other loops: with high number of lines passing
4. Add leaf-leaf, leaf-terminal and terminal-terminal lines to existing pool
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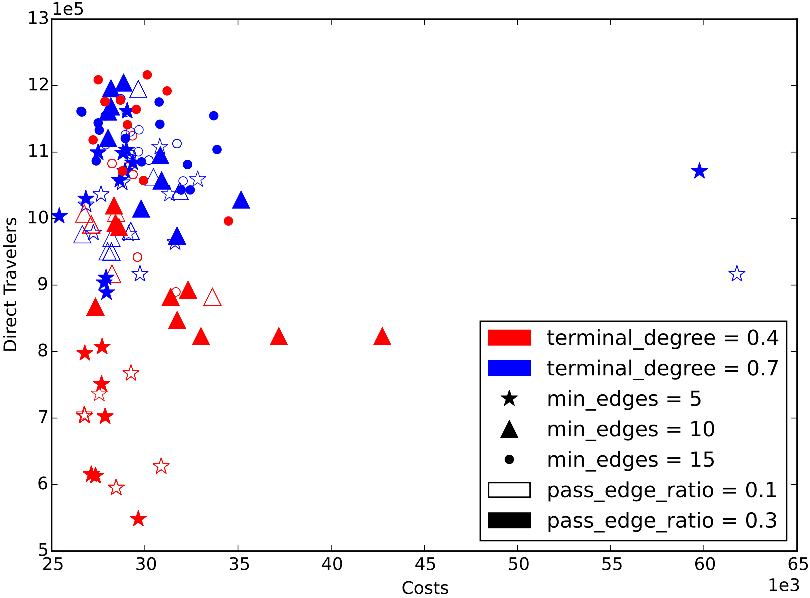
Important Parameters

- `terminal_degree`: deviation from max degree to label node 'terminal'
- `pass_edge_ratio`: Percent of edges, ranked by passenger weight, which are preferred in computation of MST
- `min_edges`: Minimal number of edges per line
- `min_distance`: Minimal euclidean distance between start- and endstation

Computation Times

		Computation Time	
	# lines	LP-Cost	LP-Direct
$K-$shortest paths	$>$ 4000	several days	several days
LPool Algo	200	1.5s	11.3s
	700	1.7s	13.4s

Results - Bahn



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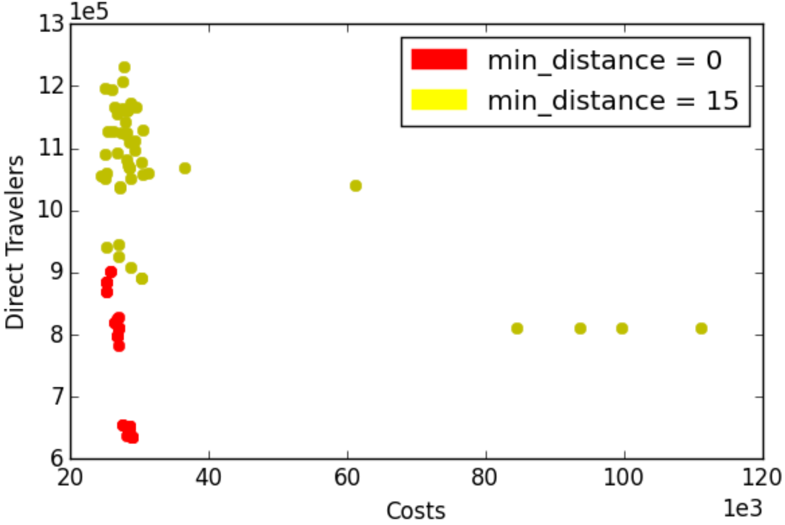

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Thanks for the attention!

Bibliography

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